NPS-OR-04-009

NAVAL
POSTGRADUATE
SCHOOL

MONTEREY, CALIFORNIA

OR MODELING OF A CONCEPTUAL SYSTEM OF SYSTEMS FOR MARITIME LITTORAL DOMINANCE IN 2020

by

Robert Andrew Berner
Ofer Gonen
Wong Chin Han
Kenneth Monfore
Tan Peng Soon
Project Advisor: LCDR Russell Gottfried

September 2004

Approved for public release; distribution is unlimited.

Prepared for: Temasek Defense Systems Institute
Block E1, #05-05, 1 Engineering Drive 2
Singapore 117576

NAVAL POSTGRADUATE SCHOOL
MONTEREY, CA 93943-5000

RDML Patrick W. Dunne, USN
Superintendent

Richard Elster
Provost

This report was prepared for and funded by the Temasek Defense Systems Institute, Block E1, #05-05, 1 Engineering Drive 2, Singapore 117576.

This report was prepared by:

RUSSELL GOTTFRIED, LCDR, USN
Military Instructor, Operations Research

ROBERT ANDREW BERNER
Operations Research

OFER GONEN
Operations Research

WONG CHIN HAN
Operations Research

KENNETH MONFORE
Instructor of Operations Research

TAN PENG SOON
Operations Research

Reviewed by:

LYN R. WHITAKER
Associate Chairman for Research
Department of Operations Research

Released by:

JAMES N. EAGLE
Chairman
Department of Operations Research

LEONARD A. FERRARI, Ph.D.
Associate Provost and Dean of Research

REPORT DOCUMENTATION PAGE			*Form Approved OMB No. 0704-0188*
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington, DC 20503.			
1. AGENCY USE ONLY *(Leave blank)*	**2. REPORT DATE** September 2004	**3. REPORT TYPE AND DATES COVERED** Technical Report	
4. TITLE AND SUBTITLE: OR Modeling of a Conceptual System of Systems for Maritime Littoral Dominance in 2020			**5. FUNDING NUMBERS** BORZ7
6. AUTHOR(S) Tan Peng Soon, Robert Andrew Berner, Ofer Gonen, Wong Chin Han, Kenneth Monfore, and Project Advisor: LCDR Russell Gottfried			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000			**8. PERFORMING ORGANIZATION REPORT NUMBER** NPS-OR-04-009
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) Temasek Defense Systems Institute Block E1, #05-05, 1 Engineering Drive 2 Singapore 117576			**10. SPONSORING / MONITORING AGENCY REPORT NUMBER**
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.			
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.			**12b. DISTRIBUTION CODE**

13. ABSTRACT *(maximum 200 words)*

This study focuses on Operations Research (OR) modeling and analysis conducted in support of a campus-wide integrated project for developing a conceptual system of systems to establish maritime dominance in the year 2020. The study features models supporting unmanned system communications, detection, and sensor fusion designs to support this mission. The paper discusses problem definition, solution methodologies—including optimization, simulation, and analytical modeling—and insights.

14. SUBJECT TERMS Unmanned Vehicles; Distributed Sensors; Sensor Fusion; Detection Modeling; Underwater Sensor Communications Grid			**15. NUMBER OF PAGES** 114
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT Unclassified	**18. SECURITY CLASSIFICATION OF THIS PAGE** Unclassified	**19. SECURITY CLASSIFICATION OF ABSTRACT** Unclassified	**20. LIMITATION OF ABSTRACT** UL

OR Modeling
of a
Conceptual System of Systems
for
Maritime Littoral Dominance in 2020

By

TDSI Operations Research Group

Robert Andrew Berner

Ofer Gonen

Wong Chin Han

Kenneth Monfore

Tan Peng Soon

August 2004

Project Advisor: **LCDR Russell Gottfried**

TABLE OF CONTENTS

ABSTRACT vi
I. INTRODUCTION 1
II. COMMUNICATIONS EFFORT 4
1. INTRODUCTION 4
1.1 Background Information 4
2. PROBLEM STATEMENT 5
3. METHODOLOGY 9
3.1 Model Formulation 9
3.1.1 Hexagonal Zones 9
3.1.2 Decision Variables 11
3.1.3 Assigning Military Values 12
3.1.4 NO GO Zones 12
3.1.5 Limitation on Assets Availability 12
3.1.6 Within Communication Range 13
3.2 Measure of Effectiveness 13
3.2.1 Objective Function 13
4. ANALYSIS AND RESULTS 14
4.1 Scenarios 14
4.2 Scenario One 15
4.3 Scenario Two 15
4.4 Scenario Three 16
4.5 Scenario Four 16
4.6 Scenario Five 17
4.7 Scenario Six 17
4.8 Scenario Seven 18
5. INTEGRATED PROJECT SPECIFIC SCENARIOS 19
5.1 Integrated Project Specific Scenario Alpha 19
5.2 SEA-5 Integrated Project Specific Scenario Bravo 23
6. CONCLUSION 26
7. SOURCES OF ERROR 27
8. FUTURE RESEARCH 27
III. FOCUSED AREA SEARCH AND IDENTIFICATION STUDY 28
1. INTRODUCTION 28
2. PROBLEM STATEMENT 28
3. METHODOLOGY 30
3.1 Model 31
3.1.1 Assets 32
3.1.2 Targets 34
3.1.3 Area 34
3.1.4 Sensor Range 34
3.1.5 Flight Patterns 35
3.1.5.1 Offset Pattern 36
3.1.5.2 Alternate Pattern 37
3.1.5.3 Diminishing Square Pattern 38

3.1.6 Tactics Involving Multiple UAVs 38
3.1.6.1 Formation Tactic 39
3.1.6.2 Sector Tactic 39
3.1.6.3 Converging Tactic..40
3.2 Measures of Effectiveness 41
4. RESULTS AND ANALYSIS 43
5. CONCLUSIONS... 47
6. POTENTIAL SOURCES OF ERROR 47
7. FUTURE RESEARCH 48
IV. SENSOR DATA FUSION MODEL 49
1. INTRODUCTION AND SUMMARY 49
2. RECOGNIZED MARITIME PICTURE (RMP) 49
3. PROBABILITY OF DETECTION AND FALSE ALARM RATE 49
4. HIGH THREAT AND LOW THREAT SCENARIOS 51
5. THE USER AS A THRESHOLD 51
6. SENSOR FUSION ADVANTAGES AND PROTOCOLS 52
7. RELEVANT SCENARIOS 54
8. ASSUMPTIONS 54
9. SIMPLE MODEL I – COOPERATIVE SEARCH AND IDENTIFICATION........ 55
10. SIMPLE MODEL II – LOW OBSERVABLE TARGET 55
11. SENSITIVITY ANALYSIS 60
12. CONCLUSIONS 61
13. FUTURE WORK 62
APPENDIX I – ACRONYM LIST 63
APPENDIX II 64
APPENDIX III.A 92
APPENDIX III.B 94
APPENDIX III.C 95
APPENDIX III.D 96
APPENDIX III.E 97
APPENDIX III.F 98
APPENDIX III.G 99
APPENDIX III.H 100
APPENDIX IV.A – THE EO SENSOR FOR THE INFLUENCE DIAGRAMS 101
APPENDIX IV.B – SENSITIVITY ANALYSIS 102
APPENDIX IV.C – COMPARING QUALITY AND QUANTITY 106

ABSTRACT

This study focuses on Operations Research (OR) modeling and analysis conducted in support of a campus-wide integrated project for developing a conceptual system of systems to establish maritime dominance in the year 2020. The study features models supporting unmanned system communications, detection, and sensor fusion designs to support this mission. The paper discusses problem definition, solution methodologies—including optimization, simulation, and analytical modeling—and insights.

I. INTRODUCTION

In October 2003, a Naval Postgraduate School Integrated Project Team (IPT) received tasking to develop a conceptual system of systems (SoS) solution for establishing maritime dominance in the littoral regions. The IPT consisted of a Systems Engineering Team and a team from the Temasek Defense Systems Institute (TDSI), a joint program of the National University of Singapore and the Naval Postgraduate School. The TDSI Team included a Communications Group, a Mechanical Engineering Group, a Physics-Sensors Group, an Information Assurance Group, and an Operations Research (OR) Group. These groups are depicted in Figure 1.

This report focuses on Operations Research (OR) modeling and analysis conducted in support of the campus-wide integrated project. While the IPT included systems engineers, mechanical engineers, sensor physicists, communications engineering designers, and information security specialists, the OR effort provided broader operational insights. Given the broad sweep of the maritime dominance problem, the OR team focused its modeling to support design decisions made by the Communications Group and the Systems Engineering Team.

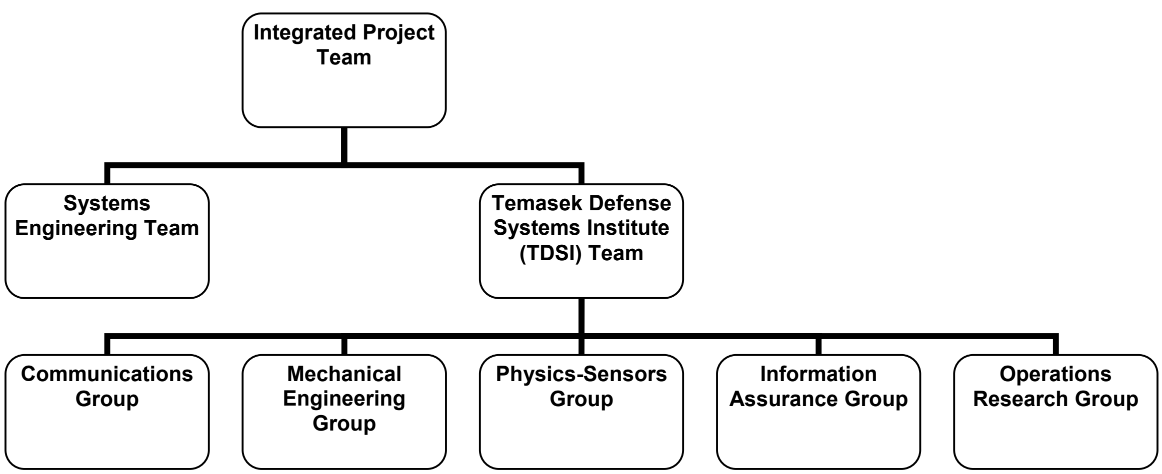

Figure 1. Integrated Project Team Organization.

Addressing Littoral Maritime Dominance in the year 2020 encompasses all areas of Maritime operations—not only air, surface, and subsurface warfare, but also support areas such as communications and data networking. Since the field of OR has tools applicable to each of these areas, the initial task appeared to require very high-level

modeling with limited insight into design-level conclusion. The OR Group sought to generate specific insights to enable resource allocation decisions.

As a starting point, the OR Group focused its efforts in two critical areas of the Maritime Dominance problem: detection and identification. More specifically, the group attempted to account for every event that needs to occur from time zero to the time of positive identification, in order to find elements that required modeling. Initial attempts to dissect this problem with decision trees and influence diagrams were thought provoking, and yielded some new ways of thinking and modeling with the Systems Engineering Team, but the magnitude of the problem quickly became very complex. Given a tight timeline, several specific elements were identified as crucial to the SoS. Ultimately, the group focused its efforts on:

1. Communications. The problem is to determine the optimal mix of communications nodes needed in the strategic, command, and tactical grids, as proposed by the TDSI Communications Group. The group provided information and studies on the impact and constraints due to variations in bandwidth and power requirements with respect to availability and operational ranges. The OR study merged this technological challenge with operational constraints imposed by the systems engineering scenario.

2. Focused Search and Detection. The OR Group used the IPT high-level unmanned aerial vehicle (UAV) definitions and sensor capabilities to make direct recommendations on which sensor capabilities and UAV types were best suited for the detection and identification missions applicable to the Integrated Project scenario. The group conducted a comparison of search methods and numbers of UAVs through a discrete event simulation software package.

3. Data Fusion. This study explored the advantages of using multiple sensors together. The study focuses on performance against a low observable target, using two and three sensors as a point of departure. The output of this study shows how the probabilities of detection (and of a false alarm) change with the number of sensors, in an effort to see how many sensors are needed, what type of sensors to employ (one or two high quality

sensors versus several low quality sensors), and the systems level trade-offs between sensor quality and data correlation processing.

The OR Group's analysis yielded insights into these specific issues applicable to the IPT's overall goal of designing a conceptual SoS architecture to establish Maritime Dominance in the Littorals. As communications, focused search and detection, and sensor fusion are crucial processes within this operating system, these models proved useful to the entire project.

II. COMMUNICATIONS EFFORT

1. INTRODUCTION

Seaweb is an advanced underwater communication network concept. The Temasek Defense Systems Institute (TDSI) Communications Group proposed this concept to support the Integrated Project's System of Systems solution to the problem of Littoral Maritime Dominance in 2020. It is an adaptive wireless network to connect sensors, tactical platforms, and remote decision makers. The goal is to take advantage of Seaweb's capability, specifically in scenarios with limited resources (number of nodes, transmission ranges) and ground deployment constraints (heavily mined areas). The problem is to configure and deploy the network in order to achieve maximum coverage as defined by the Integrated Project Team (IPT).

1.1 Background Information

In the future, underwater communication networks may acoustically link ocean bottom sensor nodes, autonomous underwater vehicles (AUVs), and surface gateway nodes to provide radio communication networks to surface units. This is the concept behind Seaweb Underwater Acoustic Networking, an idea that extends experiments in telesonar underwater acoustic signaling and ranging technology for undersea wireless network applications.

Seaweb consists of multiple deployed telesonars that form an underwater acoustic link system. This underwater "acoustic backbone" networks undersea vehicles and surface units, such as ships or floating platforms that are also equipped with the telesonars. Each of these telesonars can act as nodes that can strengthen and relay acoustic signals to radio communications (RADCOM) nodes.

Surface gateways, RADCOM units, link the undersea network to the surface RADCOM. These links can be to surface units, aerial units such as unmanned aerial vehicles (UAVs) or manned aircraft, and terrestrial units such as Low Earth Orbit (LEO) satellites. Figure 2 depicts a basic schematic of this network.

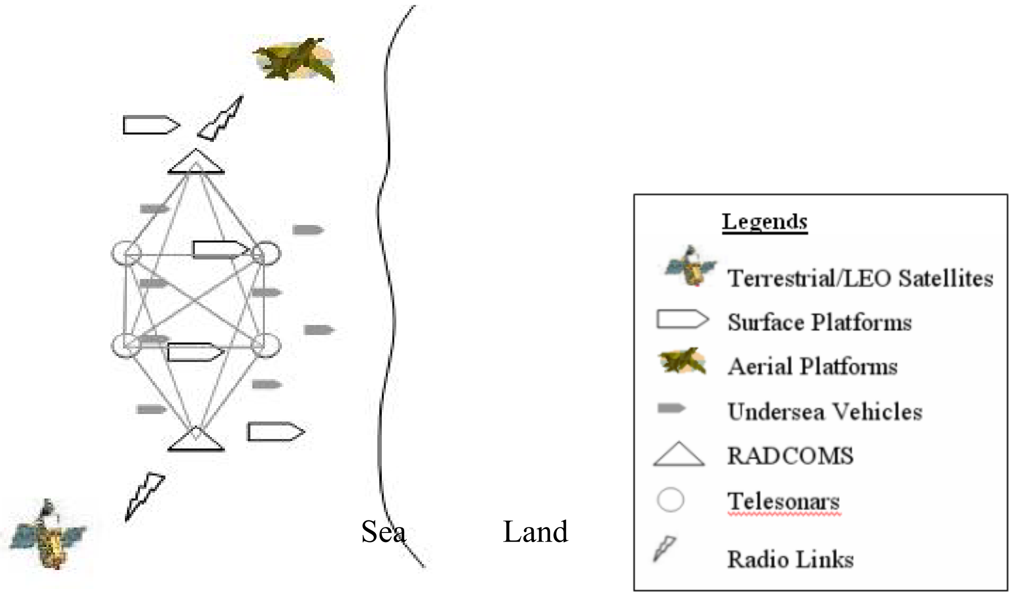

Figure 2. Seaweb concept.

Multihop topology forms the basis of the Seaweb network. Multihop peer-to-peer networks use communication links among neighboring nodes. As a result, the number of nodes deployed (i.e., the number of telesonars) determines the possible size and range of the network. Multihop peer-to-peer topology minimizes energy consumption[1]. This is especially important, as it reduces the need for battery replacement in the wireless modem of the deployed RADCOM nodes and telesonars repeaters—thus saving time and effort.

Based on Seaweb design concept, the network performs node identification, clock synchronization, geo-location, assimilation of new nodes, and self-healing following node failures. This allows Seaweb to route information packets effectively throughout the network with its distributed routing algorithms[2].

2. PROBLEM STATEMENT

The speed of sound in water is approximately 1,500 m/s. Therefore, the propagation time for an underwater signal between two nodes 10 km (current communication limitation) apart is 6.7 seconds. This geometry requires 29 hops to

[1] E.M. Sozer, M. Stojanovic, and J.G. Proakis, "Underwater Acoustic Networks," IEEE J. Oceanic Eng., Vol. 25, January 2000, pp. 72-83.

[2] For more details on Seaweb, please refer to the technical report provided by the Communications Engineering Group, affiliated with the Integrated Project.

transmit a signal along the farthest path in an 80 NM x 100 NM area, the size of the area of interest (AOI) specified by the Integrated Project scenario. This accounts for traversing the network over the longest distance in the area provided. In Figure 3, the solid line indicates the furthest possible distance between two points in the scenario area of operations (AO). The dotted lines indicate one possible shortest path (in terms of number of hops). Other paths are possible, but the minimum number of hops is 29. The distance between the centers of each hexagon in Figure 3 represents the maximum communication range (currently 10 km) of each node. RADCOM nodes are prohibited from deployment in the shaded hexagons.

In this information warfare age, stated goals are to maintain tactical advantage over the enemy using Command, Control, Communications, and Intelligence (C4I) systems to transfer a large amount of data over long distances in very short time periods. The time for a signal to traverse 29 hops is 3 minutes 14 seconds (29 x 6.7 seconds). The IPT wants to improve this for their future architectures.

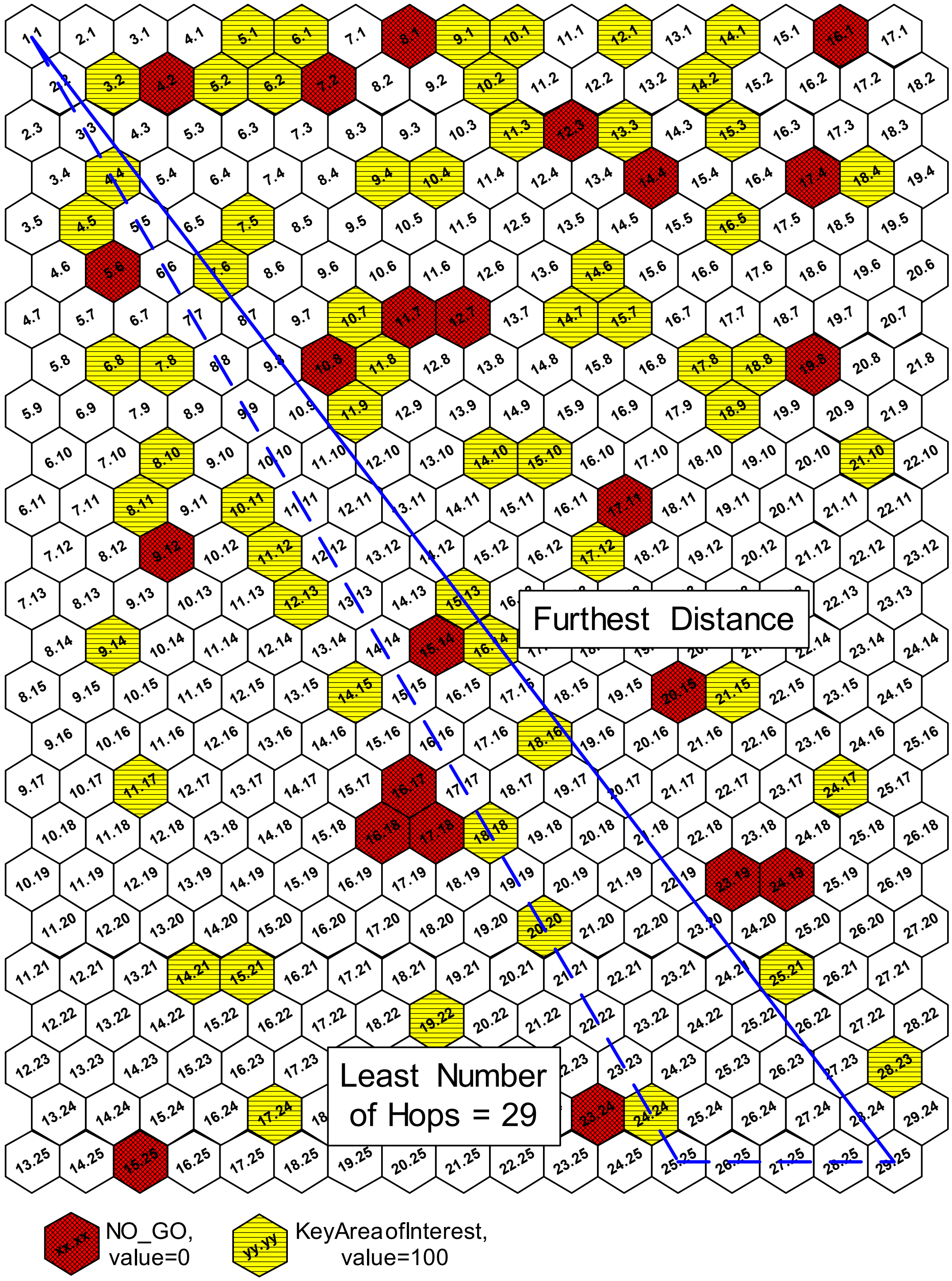

Figure 3. Two furthest points in AO.

This Operations Research (OR) Group study investigates the positioning of RADCOM nodes to communicate with the underwater acoustics nodes and send information above water. Since the speed of light in air is about 200,000 times faster than the speed of sound in water, the RADCOM nodes would transmit the data from end to end in a matter of split seconds, even if multiple hops are required for relay purposes.

In an ideal scenario, with an unlimited amount of RADCOM nodes available for deployment, a RADCOM node would be placed with every single underwater acoustics node. This would result in a perfect communications network of 425 RADCOM nodes. But this is neither possible nor practical from an operational perspective. With such a great number of nodes, the cost to field such a system would be prohibitive, coupled with the increased technical complexity of hardware and software protocols.

An alternative approach is for each RADCOM node to support multiple fixed and mobile underwater acoustics nodes, within communications range constraints. Since each RADCOM node has limited communications range, the problem is to minimize the number of RADCOM nodes required for complete coverage of all key areas of military interest. In the event that the number of RADCOM nodes is fixed, the problem is further defined as the optimal node placement in order to minimize the amount of uncovered area.

Over and above the resource and range limitations, there are other constraints on the deployment. For example, there are areas of interest that could face threats, such as mines, which preclude the placement of RADCOM nodes, especially if the RADCOM functions are to be performed by surface vessels. However, these areas still require communication coverage, posing additional restrictions on the optimal number of nodes required. In the event that the number of available RADCOM nodes is limited, there is still a need to find an optimal placement strategy such that the uncovered areas are minimized.

Taking these goals, requirements, and restrictions into consideration, the problem statement for the OR study is simplified in the following way:

- Minimize the number of RADCOM nodes required for a given area of coverage;
- By choice of optimal placement of these nodes;
- Given the tactical environment; and
- Subject to asset availability and Radio Frequency (RF) limitations.

3. METHODOLOGY

The model divides the AO into hexagonal zones and represents the AO as a rectangular grid. Each zone has a tactical value of 0 to 100 based on military importance, enabling development of a linear program in General Algebraic Modeling System (GAMS) that penalizes lack of coverage by these values. The objective is to minimize the total value or sum of penalties in tactically important areas that lack communications. The model provides the optimal number and placement of RADCOM nodes in order to provide complete coverage to all areas of interest. The various architectures being considered by the IPT limit the number of vessels to between 20 and 30. Cost, logistical support, and tactical deployment considerations also limit the number of nodes that may be fielded in a realistic environment. Asset availability constraints are an aspect of the model to determine optimal placement for RADCOM nodes in different resource restricted scenarios.

Initial analysis uses a generic scenario. Variations on this scenario include imposing more restrictions to ground deployment, increasing the areas of interest, and increasing the communication range capability. After obtaining some results with this generic model, the OR Group uses the Integrated Project's specific scenarios to obtain directly applicable results.

3.1 Model Formulation

3.1.1 Hexagonal Zones

This model represents the AO using hexagonal grids. The centers and corners of grids depict possible locations for the underwater acoustics nodes. The most efficient

way to pack circles of equal sizes is to have each one surrounded by six others[3], as seen in Figure 4.

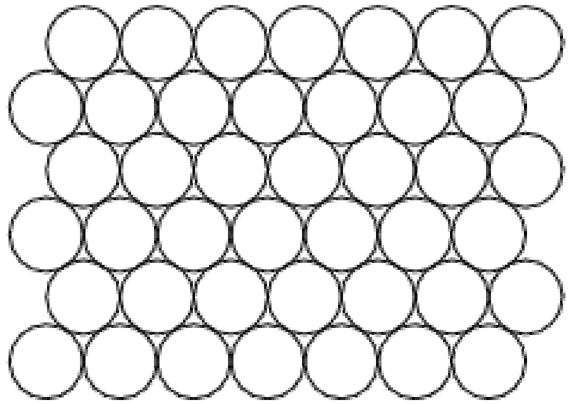

Figure 4. Hexagonal circle packing.

However, since a great deal of area is uncovered in the space between circles, the circles must be "squeezed" into regular hexagons in Figure 5. The center of each zone has a unique set of coordinates to represent its location.

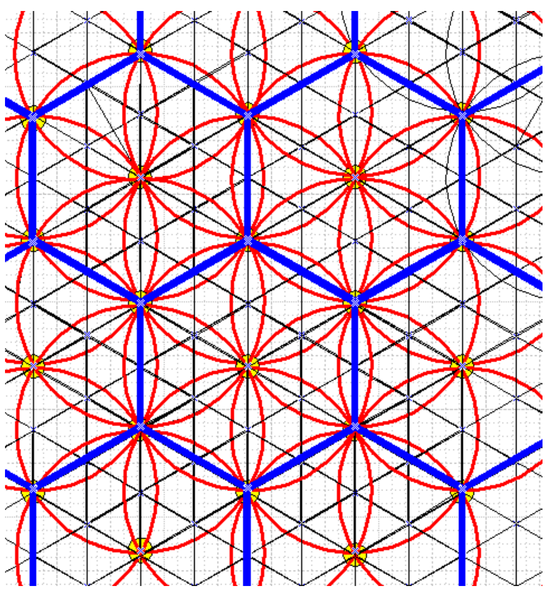

Figure 5. Hexagonal zones.

[3] Amy C. Edmondson, "Tales Told by the Spheres: Closest Packing," Chapter 8, pp. 102-106, http://www.angelfire.com/mt/marksomers/96.html; http://icl.pku.edu.cn/yujs/MathWorld/math/c/c322.htm.

The distance between the centers of each hexagon is 10 km, the communication capability of the underwater acoustics nodes specified by the Communications Group.

The northwest quadrant of the AO shows the coordinate system in Figure 6.

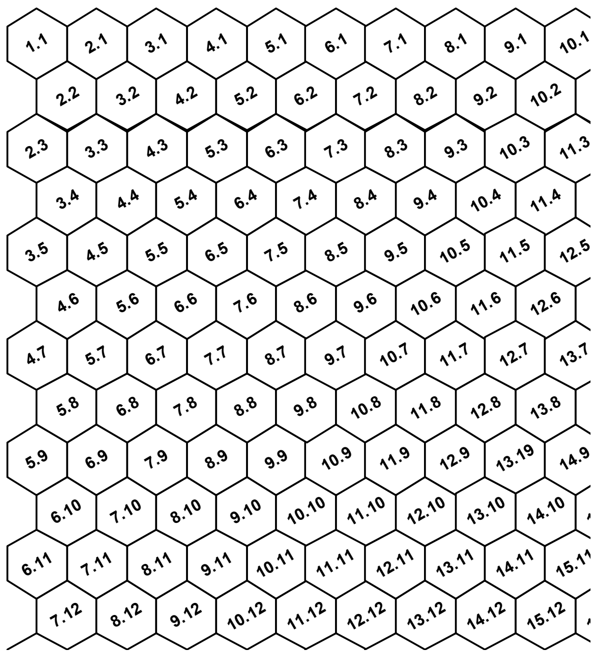

Figure 6. The generic grid, northwest quadrant.

3.1.2 Decision Variables

The primary decision variables in the model are

X(i, j) = 1, if a RADCOM node is to be deployed in zone (i, j)
= 0, otherwise.

The secondary decision variables in the model are

Y(i, j) = 1, if zone (i, j) has no communication coverage,
= 0, otherwise.

The pair (i, j) indicate the coordinates of this AO, where i = {1,2, …, 29} and j = {1,2, …, 25}.

3.1.3 Assigning Military Values

Zones receive values ranging from 0 (no military interest) to 100 (highest priority) to signify their tactical importance. In the event of no communication coverage, these values represent penalties. This identifies the key areas of interest and provides a means of weighting relative importance or tactical value.

3.1.4 NO GO Zones

The model represents the restriction that certain zones are not allowed for deployment of RADCOM nodes by referring to such areas as "NO GO" zones. Recalling that X(i, j) = 0 if no RADCOM node is deployed in zone (i, j), the following constraint equation serves as an example of how to prevent the use of various off-limit zones (from the example in Figure 3):

X("8","1") + X("16","1") + X("4","2") + X("7","2") + X("12","3") + X("14","4") + X("17","4") + X("5","6") + X("11","7") + X("12","7") + X("10","8") + X("19","8")
+ X("17","11") + X("9","12") + X("15","14") + X("20","15") + X("16","17") + X("16","18") + X("17","18") + X("23","19") + X("24","19") + X("23","24") + X("15","25") = 0.

The equation forces the value of each zone to 0, by making their sum 0.

3.1.5 Limitation on Assets Availability

Resource limitations place an upper bound on communications nodes. The following constraint equation models the resource limitations on the number of RADCOM nodes available for deployment:

Sum((i, j), X(i, j)) ≤ N,

where N is the number of nodes available. In effect, it is summing over all i's and j's, such that:

X(1, 1) + X(1, 2) +, ……, + X(29, 25) is less than or equal to N.

For example, in the scenarios studied by the IPT where N = 30, the sum of all zones in which a RADCOM unit is deployed cannot exceed 30.

3.1.6 Within Communication Range

Placing a RADCOM node in zone (i, j) effectively provides communication coverage to underwater acoustics nodes in the six adjacent zones within the communication range of 10 km. That is, within range,

X(i-1, j-1) + X(i, j-1)	{2 adjacent nodes north of X(i, j)}
+ X(i-1, j) + X(i, j) + X(i+1, j)	{2 adjacent nodes to the east and west of X(i, j)}
+ X(i, j+1) + X(i+1, j+1)	{2 adjacent nodes south of X(i, j)}
+ Y(i, j) ≥ 1;	{Y(i, j) either has (= 0) or does not have (= 1) coverage}

Mathematically, this means that if there is no RADCOM node in grid (i, j) and all six adjacent zones, then Y(i, j) must be set to 1, signifying that zone (i, j) does not have any communication coverage. Y(i, j) may be set to 0 or 1 if there is a RADCOM node at any of the position[4]. Since the objective is to minimize the sum of values of the uncovered areas, Y(i, j) will be set to 0 whenever possible.

In summary, the three constraints are represented algebraically as:

$$\sum X_{i,j} = 0, \forall i, j \in NO\_GO \tag{1}$$

$$\sum X_{i,j} \leq N, \forall i, j \tag{2}$$

$$X_{i-1,j-1} + X_{i,j-1} + X_{i-1,j} + X_{i,j} + X_{i+1,j} + X_{i,j+1} + X_{i+1,j+1} + Y_{i,j} \geq 1, \forall i, j \tag{3}$$

3.2 Measure of Effectiveness

3.2.1 Objective Function

The goal is to cover all tactically important zones in the AO. The corresponding objective is to minimize the sum of military values of the uncovered zones. The equation for this objective function is:

Z = Sum((i, j), Y(i, j)*importance(i, j)),

[4] This technique is similar to example in Ronald L. Rardin, "Optimization in Operations Research," Prentice Hall, 1998, pp. 569-571.

where importance (i, j) is the tactical significance of the zone at coordinate (i, j). By summing over all i's and j's, the objective function adds all the military values of zones (i, j) that have values of Y(i, j) set to 1. Recalling the range of values for Y(i, j), there is no communication coverage for that particular zone.

The presence of communications, signified by Y(i, j) = 0, does not contribute to this objective. While it is desired to achieve an objective function value of 0, where all areas of interest (which are given a positive value other than 0) are covered, the limitations and restrictions make this infeasible.

4. ANALYSIS AND RESULTS

4.1 Scenarios

The baseline scenario assigns either 100 or 0 to distinguish between AOI and non-AOI, respectively. Twenty-three NO GO zones are setup and 30 RADCOM nodes are made available for deployment. Subsequent scenarios build on this baseline scenario by varying the military values, the number of NO GO zones, or the communication range to gain insight and perform some "what-if" analysis.

Table 1 summarizes the conditions used for the generic scenarios:

Scenarios	Military Values	NO GO Zones	Comms Range	MAX_NODES
One	0 and 100 only	23 zones	10 km	30
Two	0 and 100 only	2 x 23 zones	10 km	30
Three	0 and 100 only	23 + 59 (key areas of interest) zones	10 km	30
Four	0, 70, and 100	23 zones	10 km	30
Five	0, 30, 70, and 100	23 + 12 (key areas of interest) zones	10 km	30
Six	30, 40, 50, 60, 70, 80, 90, and 100	23 zones	10 km	30
Seven	0, 70 and 100	23 zones	20 km	30

Table 1. Generic scenarios.

The primary purpose of flexing the model to all of these basic scenarios serves to validate its performance. After running through this set, we can see if the solutions conform to expectations. A computational and pictorial formulation of the basic scenario is in Appendix II.

4.2 Scenario One

This is the baseline scenario (using the grid established in Figure 3). Variations to the conditions in this scenario form secondary scenarios, which are used for subsequent sensitivity analysis.

Scenario One assigns military values of either 100 or 0 to AOI and non-AOI, respectively. The communication range is set to 10 km. There are 23 NO GO zones and 30 RADCOM nodes available for deployment.

The model returns an objective function value of 300, indicating it is infeasible to provide total coverage under these conditions. There are three zones of value 100, each not having communication coverage. To find the number of nodes required for complete coverage, the value of MAX_NODE is increased by 1 unit for each run until the objective function value reaches 0. It is noted that 33 nodes are required in this case. For details of optimal node placement and GAMS implementation, refer to Appendix II. The remaining scenarios form a "what-if" analysis, changing operational attributes to obtain insight into the feasible region for this communication grid under varying tactical scenarios. After analysis of these alternatives, this study uses the validated model to provide insight for the Integrated Project.

4.3 Scenario Two

This scenario investigates the effects of doubling the number of NO GO zones. It also assigns military values of either 100 or 0 to AOI and non-AOI, respectively. The communication range is set to 10 km and 30 RADCOM nodes available for deployment. There are now 46 NO GO zones, however.

The model returns an objective function value of 300, indicating it is infeasible to provide total coverage under these conditions. There are three zones of value 100 without communication coverage. To find the number of nodes required for complete coverage the value of MAX_NODE is increased by 1 unit for each run until the objective function value reaches 0. Again, 33 nodes are required in this case. The optimal number of 33 nodes is the same as that for Scenario One. Doubling the number of NO GO zones does not affect the optimal number in this case, since there are a lot of other possible locations for the nodes placement. For details of optimal node placement and GAMS implementation, refer to Appendix II.

4.4 Scenario Three

This scenario investigates the effects of imposing NO GO restriction on the key areas of interest. That is, no nodes are allowed to be deployed in our key areas of interest, where coverage is required. This is not unrealistic. In the minesweeping example, there is a premium placed on information in the vicinity of the mine, although placing a communication node there would endanger that component. To achieve this, Scenario Three assigns military values of either 100 or 0 to AOI and non-AOI, respectively. The communication range is set to 10 km and 30 RADCOM nodes available for deployment. There are 82 NO GO zones.

The model returns an objective function value of 800, indicating it is infeasible to provide total coverage under these conditions. There are eight zones of value 100 each not having communication coverage. To find the number of nodes required for complete coverage the value of MAX_NODE is increased by 1 unit for each run until the objective function value reaches 0. It is noted that 38 nodes are required in this case.

The optimal number of 38 nodes is only five more than that for Scenario One, though the number of NO GO zones more than tripled, from 23 to 82. Imposing this heavy additional constraint on deployment only affects optimality incrementally. For details of optimal node placement and GAMS implementation, refer to Appendix II.

4.5 Scenario Four

This scenario investigates the effects of assigning a value of 70 to all six adjacent zones of each of the key AOI. It is unlikely that if a zone is assigned a military value of 100, that it will be surrounded by six adjacent zones of no military interest. A more plausible case would be that these adjacent zones have some intermediate values. The communication range is set to 10 km and 30 RADCOM nodes available for deployment. There are 23 NO GO zones.

The model returns an objective function value of 4,140, indicating it is infeasible to provide total coverage under these conditions. To find the number of nodes required for complete coverage, the value of MAX_NODE is increased by 1 unit for each run until the objective function value reaches 0. Forty-six nodes are required in this case. The optimal number of 46 nodes is now 13 more than that for Scenario One. By having the six adjacent zones having intermediate military values effectively increases the area of

coverage requirement, and hence the number of nodes increases too. For details of optimal node placement and GAMS implementation, refer to Appendix II.

4.6 Scenario Five

Using Scenario Four as a baseline, this scenario investigates the effects of assigning a value of 70 to adjacent zones of each of the key AOI along a specific avenue of approach, while secondary AOI are given values of 30. Additional constraint is further added, such that no nodes are allowed to be deployed in some of the 100-point zones. The communication range is set to 10 km and 30 RADCOM nodes available for deployment. There are 35 NO GO zones.

The model returns an objective function value of 2,670, indicating it is infeasible to provide total coverage under these conditions. To find the number of nodes required for complete coverage, the value of MAX_NODE is increased by 1 unit for each run until the objective function value reaches 0. It is noted that 50 nodes are required in this case. Although the area of coverage is the same as the previous scenario, the total military value is now lower and there are a lot of low value zones. Giving different values to the same coverage areas essentially changes the priorities of the zones. With these additional constraints, the choice of locations is restricted further. More nodes are required for complete coverage. For details of optimal node placement and GAMS implementation, refer to Appendix II.

4.7 Scenario Six

This scenario investigates the effects of having the military values decrease as the distance from the land increases. Military values of 100 are assigned to the first three layers of AOI nearest to land, 90 to the next three layers, 80 to the next three layers and so on. The communication range is set to 10 km and 30 RADCOM nodes available for deployment. There are 23 NO GO zones.

The model returns an objective function value of 90, indicating it is infeasible to provide total coverage under these conditions. To find the number of nodes required for complete coverage, the value of MAX_NODE is increased by 1 unit for each run until the objective function value reaches 0. It is noted that 33 nodes are required in this case. The results suggest that although values ranging from 100 (nearest to land) to 30 are

assigned, we would need to sacrifice the three zones with lowest value of 30 each. The optimal number for complete coverage is, however, the same as the baseline case, which is effectively determined by the zones, and not really dependent on the absolute values. The optimal node placement is also the same as Scenario One. For details of optimal node placement and GAMS implementation, refer to Appendix II.

4.8 Scenario Seven

Using Scenario Four as baseline, the scenario investigates the effects of doubling the communication range to 20 km. In the event that the IPT needs to weigh the cost of technology improvement, the extended communication capability may be expected to bring about a significant reduction in the number of RADCOM nodes required.

Military values of 70 are assigned to all six adjacent zones of each of the key AOI. It is unlikely that for a zone assigned a military value of 100 will be surrounded by six adjacent zones with no military interest. As before, the more plausible case is that these adjacent zones have some intermediate values. The communication range is set to 20 km and 30 RADCOM nodes available for deployment. There are 23 NO GO zones. The WITHIN_RANGE constraint is now modified to reflect the additional 12 nearest zones that are within reach of the RADCOM node:

$$
\begin{aligned}
& X(i-2, j-2) + X(i-1, j-2) + X(i,.j-2) \\
& + X(i-2, j-1) + X(i-1, j-1) + X(i, j-1) + X(i+1, j-1) \\
& + X(i-2, j) + X(i-1, j) + X(i, j) + X(i+1, j) + X(i+2, j) \\
& + X(i-1, j+1) + X(i, j+1) + X(i+1, j+1) + X(i+2, j+1) \\
& + X(i, j+2) + X(i+1, j+2) + X(i+2, j+2) \\
& + \ Y(i, j) = G = 1;
\end{aligned}
$$

Doubling the communication range reduces the number of nodes required for complete coverage from 46 to 22, which is well within the asset availability limit of 30. For details of optimal node placement and GAMS implementation, refer to Appendix II.

Figure 7 shows the change in objective function values as the number of nodes increases for two cases of communication ranges of 10 km and 20 km, respectively:

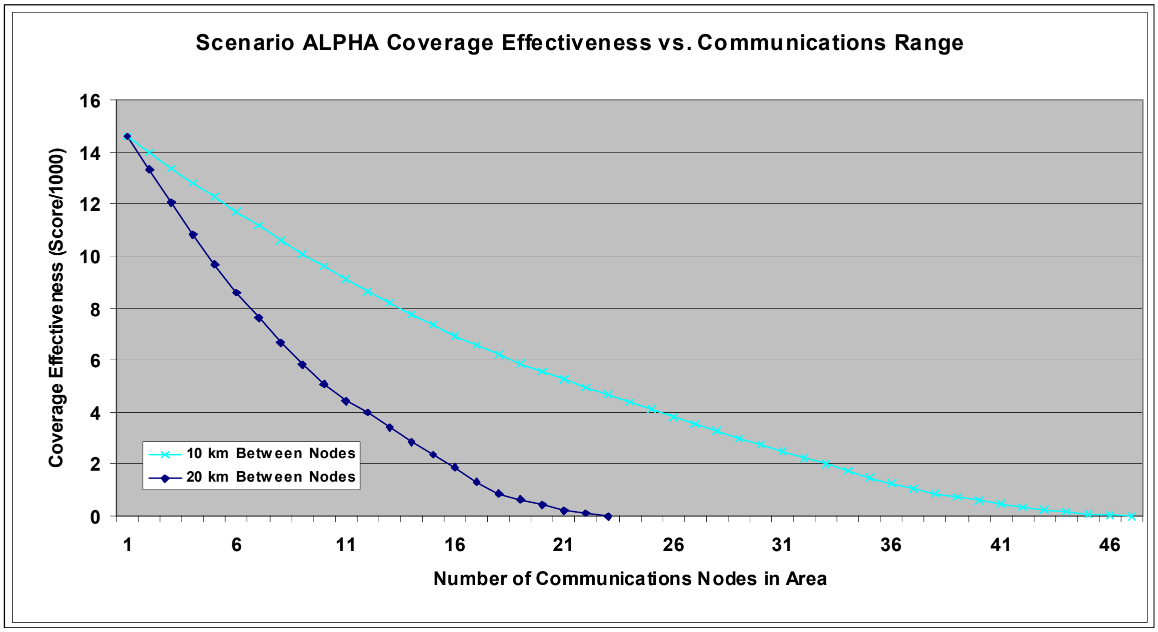

Figure 7. Effects of doubling communication range.

In both cases, the objective function values decrease at a decreasing marginal rate, which is rather intuitive. As the key AOI with higher military values are assigned RADCOM nodes to provide communication coverage, remaining nodes provide coverage to the areas of secondary interest, which have lesser values. Doubling the communication range increases the number of adjacent zones three-fold, from six zones to 18 zones. In this case, the three-fold increase in adjacent zones reduces the number of nodes by only a factor of 46/22 = 2.1.

5. INTEGRATED PROJECT SPECIFIC SCENARIOS

5.1 Integrated Project Specific Scenario Alpha

Applying these analysis techniques and methodology to the Integrated Project's specific tactical scenario, its grid layout, areas of interest, and NO GO zones, the systems engineering team sought insight into System of Systems design decisions. This scenario includes a shoreline to the northern, southern, eastern, and western boundary limits of the AO. Specific design considerations of interest include force size and capacity, approximate topology, and feasibility. Military values of 100 are assigned to the first 30 NM of water nearest to land, and 30 to all other areas. The communication range is

set to 10 km and 30 RADCOM nodes available for deployment. There are 24 NO GO zones.

The model returns an objective function value of 6,530, indicating it is infeasible to provide total coverage under these conditions. Figure 8 shows the optimal placement of the limited number of 30 nodes and indicates key areas of interest (100 points) that are not covered. To find out the number of nodes required for complete coverage, the value of MAX_NODE is increased by 1 for each run until the objective function value reaches 0. It is noted that 71 nodes are required in this case. For details of GAMS implementation, refer to Appendix II.

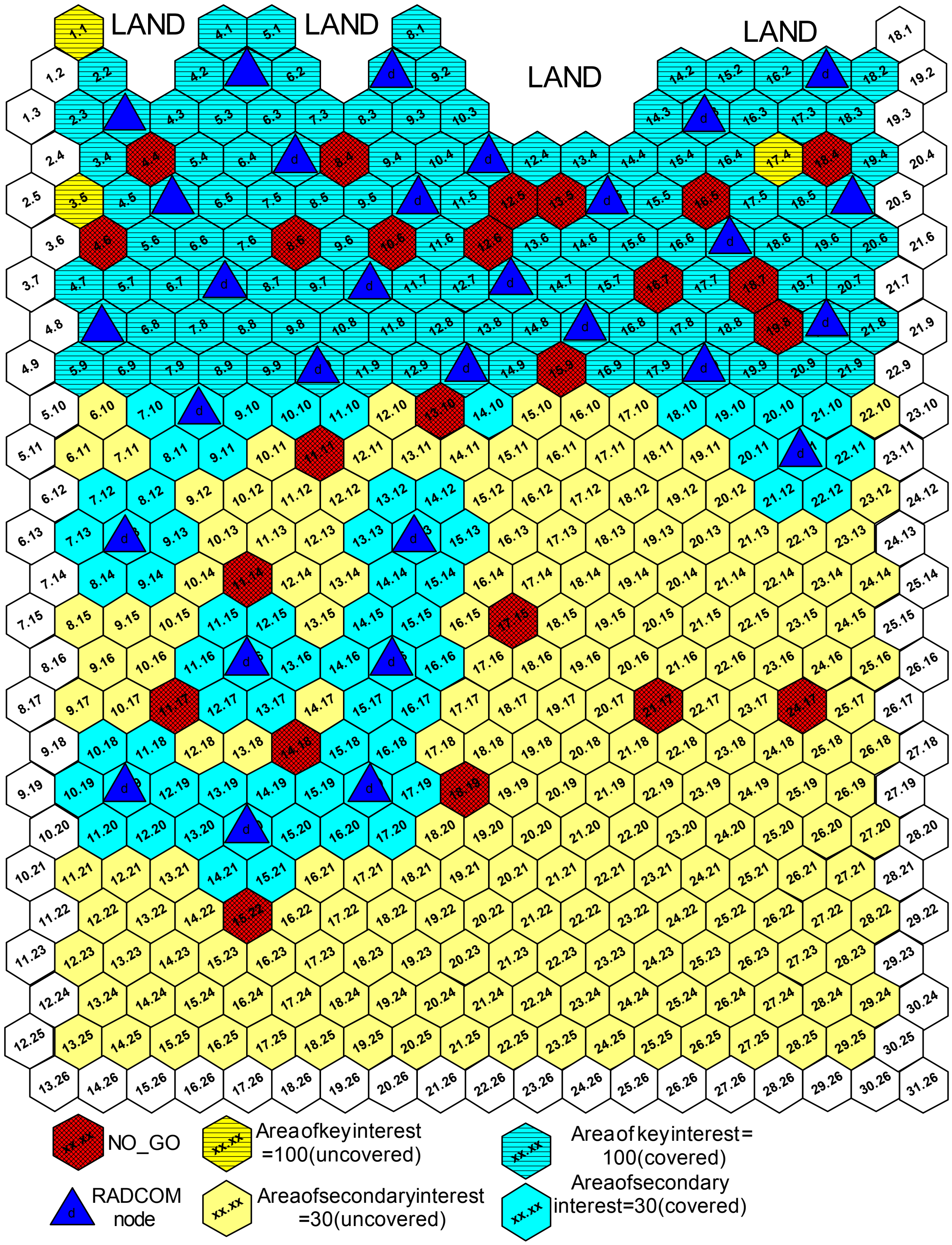

Figure 8. Node placement for Integrated Project's specific Scenario Alpha with 30 nodes.

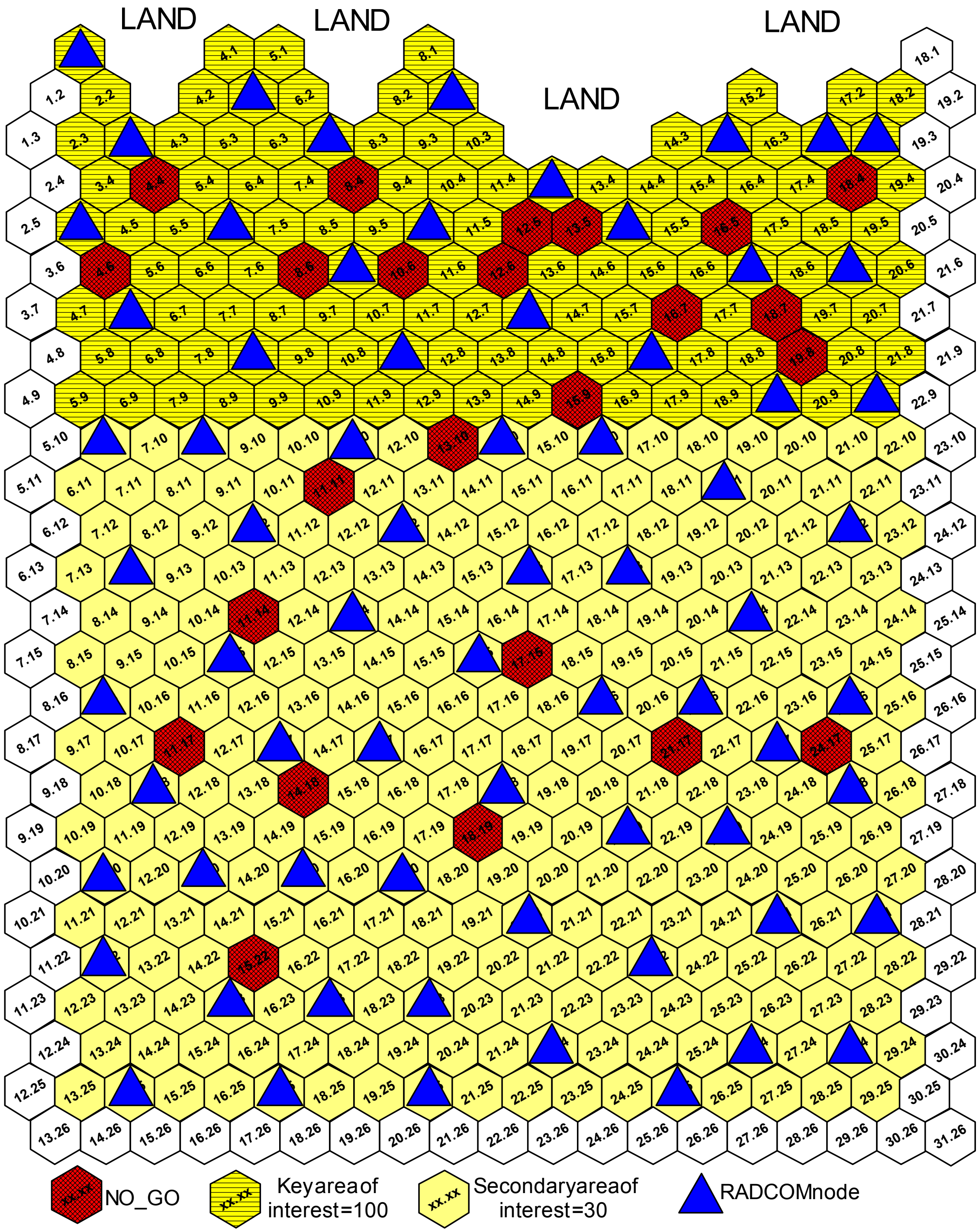

Figure 9. Node placement for SEA-5 specific Scenario Alpha with 71 nodes.

5.2 SEA-5 Integrated Project Specific Scenario Bravo

The next scenario investigates the effects of doubling the communication range. Using Scenario Alpha as a baseline, military values of 100 are assigned to the first 30 NM of water nearest to land, and 30 to all other areas. The communication range is set to 20 km and 30 RADCOM nodes available for deployment. There are 24 NO GO zones. Optimal objective function value of 0 is achieved with 27 nodes. Figure 10 shows the optimal placement of the nodes with the communication range of each RADCOM doubled.

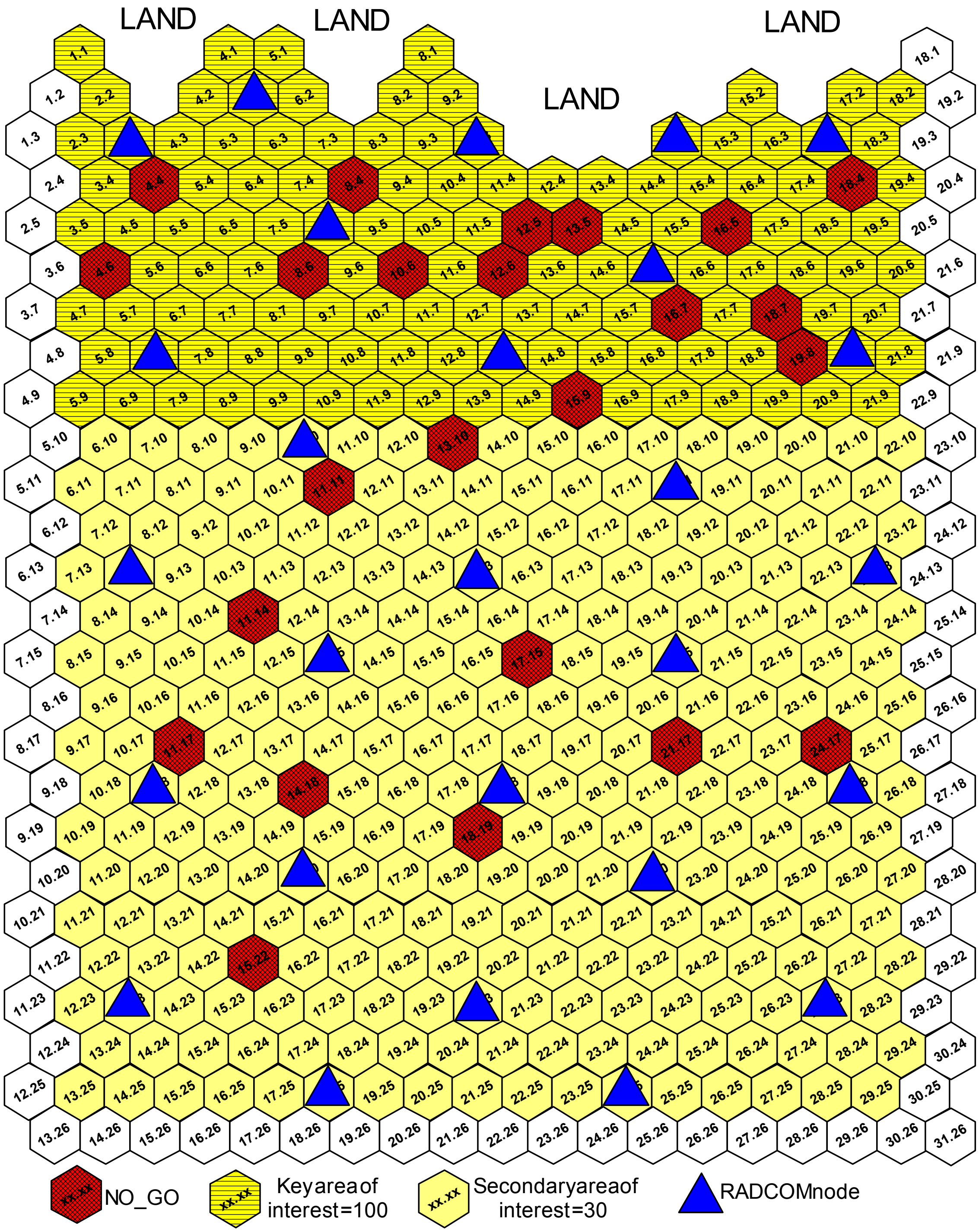

Figure 10. Node placement for SEA-5 specific Scenario Bravo.

Figure 11 compares the change in objective function values as the number of nodes available increases, for communications ranges of 10 km and 20 km, respectively.

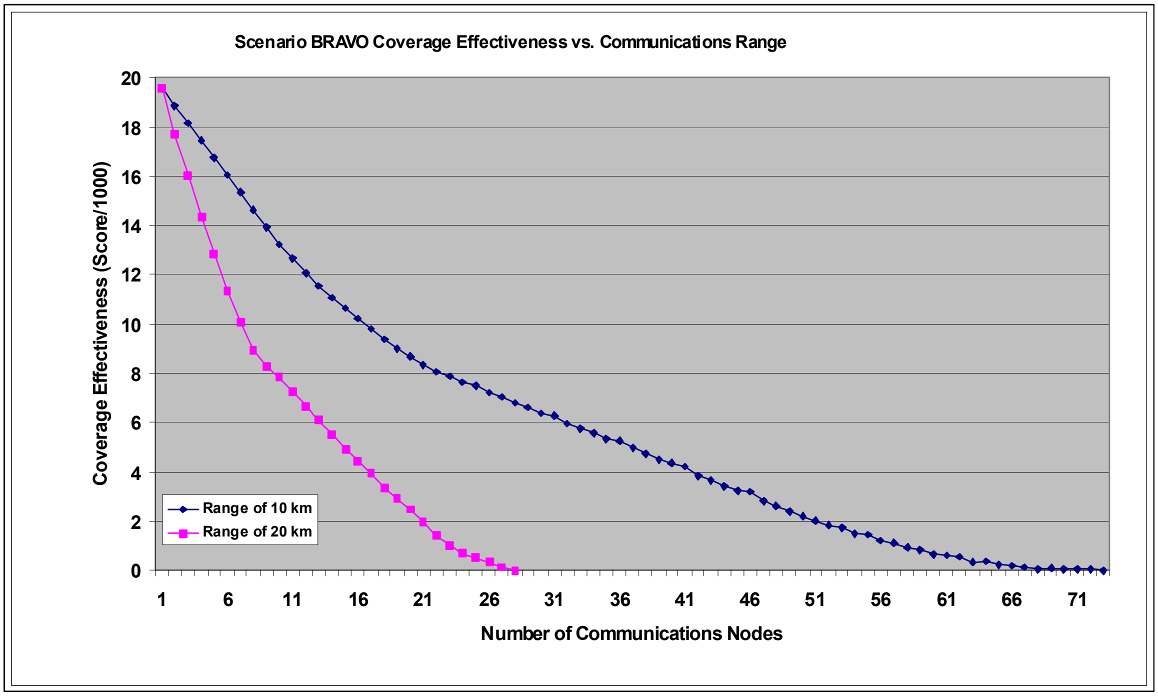

Figure 11. Effects of doubling communication range.

For both cases, there is an initial drop of steep gradient with each additional node covering key areas of tactical interest, each worth a value of 100. Another nearly constant gradient interval appears when the nodes cover those zones of 30 points each. The gradient for the 20 km range curve is steeper than the 10 km range curve since the nodes cover 18 adjacent zones. By increasing communication range by a factor of 2, there is a three-fold increase in adjacent zones (from six zones to 18 zones), and a reduction in the number of nodes required by a factor of 2.6. Also note that towards the tails of both curves, there are diminishing returns—in another words, a large number of nodes are required in order to provide complete coverage.

Half the number of required nodes provide 80% coverage. Figure 12 depicts the marginal benefits of additional communication nodes. Decision makers can determine the trade-off between doubling spending and achieving that last 20% of coverage.

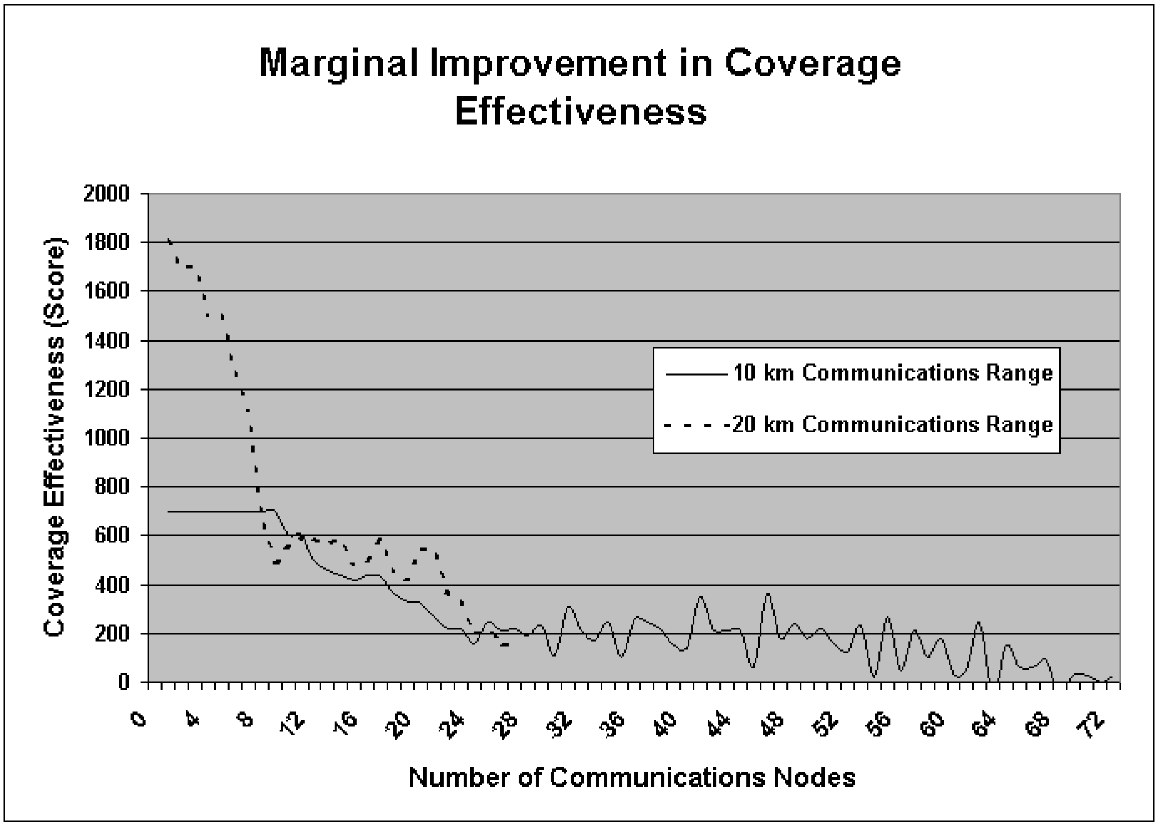

Figure 12. Marginal improvement in objective function values.

6. CONCLUSION

For the Integrated Project's tactical scenarios, our analysis suggests that an initially proposed asset availability of 30 nodes is not able to meet the total communication coverage requirement. Thirty nodes are, however, able to provide complete coverage of the 35-40 NM AOI running up to the shoreline. This leaves two-thirds of the AO uncovered. If a primary avenue of approach or "passage way" is identified, excluding the rest of the AO, optimal placement of communication nodes may be possible using only 30 nodes.

If complete AO coverage is required, then one possible solution is to increase asset availability from the current level of 30 to at least 71 nodes. Cost is definitely a consideration and the complexity of such an enormous network also increases, especially when scheduling placement, communication routing, and grid maintenance are considered.

On the other hand, research efforts are currently underway to improve communication ranges and still achieve acceptable throughput, bit-error rates, and power

consumption levels, all of which suffer as range increases. It is interesting to see that if the technology permits a doubling of communication ranges, the number of nodes required decreases from 71 to 27.

The diminishing returns of the communications subsystem also exhibit themselves in this model. The end result is that it is up to system designers to decide if the benefits of complete coverage warrant additional spending, or if an 80% communications coverage solution is acceptable.

7. SOURCES OF ERROR

Despite efforts to limit the sources of error for this study, there are some limitations associated with this model.

Importance values and avoidance zones affect optimality. Solutions are only as good as the military values that are being assigned to these zones. An underlying assumption is that there are ways to assign these values, either through intelligence, terrain analysis, operational planning, or technical inputs. Planners can effectively assign relative weights to a grid layout, which mimic as closely as possible the AO.

Communications in the model resemble a cookie-cutter function, where communications are either "covered" or "not covered." This is unlikely in real world communications. Environmental effects, terrain, water properties, and many other factors affect system performance in various ways. These include ducting, multipath, and fade phenomena that change effective communication ranges during different times of day and at different locations. The linear program does not account for these factors.

8. FUTURE RESEARCH

The model used in this study has potential for further enhancements. Its formulation is flexible enough to support irregularly shaped areas of operations or to be expanded as required. The simple model can yield insight into the problem by trading off technological and operational design decisions in an effort to obtain optimal solutions.

A possible improvement is to model communication nodes of different capabilities and provide the optimal mix for similar objective functions. The model can also include constraints on the distance between RADCOM nodes, or constraints on the minimum and maximum number of nodes for different sectors.

III. FOCUSED AREA SEARCH AND IDENTIFICATION STUDY

1. INTRODUCTION

Another Temasek Defense Systems Institute (TDSI) Operations Research (OR) Group's modeling effort involved a study on tactics and search patterns for Unmanned Aerial Vehicles (UAVs) in focused areas of interest. The OR Group used the Integrated Project Team's (IPT) high-level UAV definitions and sensor capabilities to make direct recommendations on which sensors and platforms were best suited for the detection and identification missions applicable to the project's scenario. Modeling and simulation programs, including a discrete event simulation software package, yielded basic insights into system performance and operational employment.

2. PROBLEM STATEMENT

The "time zero to the time of positive identification" model discussed in the introduction is fundamentally separated into two parts: detection and identification. These two processes are specifically categorized in the Integrated Project's system engineering functional decompositions into "Surveillance" and "Threat Analysis and Evaluation." The distinction between detection and identification is important because the areas inherently involve many different aspects of surface search. They take time, use assets, and fundamentally affect pursuit of Maritime Dominance. Although the two processes are related, each has inherent difficulties.

The problems posed by detection and identification involve the allocation of resources. Planners address issues such as the search area to be covered, the level of resolution required, and the number of assets necessary to attain that level of resolution. Obviously, the decision maker would like to know where all of the contacts are in the entire area of interest, all of the time, but this is often unrealistic. Likewise, the decision maker would like to have 100% of the targets in the entire area of interest identified with no uncertainty. This also may be unattainable. Detection and identification are often linked in that many of the same assets perform both functions, which compete for time, influencing overall availability for the other function. In other words, if a search asset must also identify all detected contacts, it needs to spend longer

amounts of time at each detected contact. This increases the overall time required to search a given area.

Current tactics employ many levels of assets to perform cooperative search and identification functions. For legacy systems, the example of an E-2C obtaining the overall search area picture is germane. The E-2C performs the detection function and relays the detection picture to other identification assets such as jet aircraft or helicopters. These assets then investigate the E-2C's picture and identify targets of interest.

In the absence of the E-2C picture, commanders rely on P-3C, S-3B, SH-60B, or shipboard radars to provide the overall surface picture. This mission, called Surface Search and Control (SSC), is vital to the commander's situational awareness. Typically, aircraft work together to provide the best picture possible. These aircraft both detect and investigate contacts with their own dedicated sensors, or they relay the position of targets of interest to the nearest available asset after initial detection. All assets then regularly update contact track information such as course, speed, and identification. Normally, during SSC missions, aircraft use radar to get an idea of what contacts are in the area and investigate as many as fuel endurance permits. If radar coverage is limited, to avoid the risk of counter detection for example, then commanders rely on passive sensors such as Forward Looking Infrared (FLIR), Electro-optical/Infrared (EO/IR), Electronic Support Measures (ESM) for detection and Identification (ID), if not on visual detection alone. More often than not, all manned assets in the area are typically performing the SCC mission.

The IPT developed two future-based architectures, in which forces rely heavily on Unmanned Vehicles (UVs) to perform the missions described above. More specifically, the team investigated the use of unmanned aerial vehicles (UAVs) in place of the E-2C, jet, and helicopter aircraft. The goal of the focused search study is to investigate this possibility. The purpose is two-fold: (1) investigate what tactics work best with UAVs, and (2) find out how various numbers of UAVs perform those tactics.

It is important to understand that the search and identification problem is easier if it is assumed that the picture of the search area (provided by the E-2C or other high altitude orbiting asset) is available on demand. If such an asset exists, then UAV tasking essentially resolves itself into a "traveling salesman problem." In this problem, the

salesman is given a finite number of cities, along with the cost of travel between each pair of them. The challenge is to find the least expensive route to visit all cities and return to the starting point. This type of problem can be solved with optimization techniques or heuristics.

For the Maritime Dominance problem, the traveling salesman problem is the best way for UAVs to execute identification missions. However, it is complicated by the fact that the sites for visitation move and that numerous environmental effects, such as wind and weather, affect UAV flight paths. The relative motion between contacts also changes optimal routing and sensor performance affects dwell time at each vessel.

Models of this problem exist (Washburn's SSC model[5]), however, typical solutions make the critical assumption that the contact picture is already known in advance. This is a major assumption. If the E-2C is not present, and the overall surveillance picture is not available, the complete search area picture may not always exist. Even if it does exist, the level of resolution may likely not be adequate to enable identification of all targets in the area with certainty.

This study investigates the circumstance in which a complete surface area contact picture does not exist. The question is how to employ the search assets in the area of interest using which tactics and in what numbers.

3. METHODOLOGY

In order to study how UAVs might be used in the detection and identification mission, the team decided to simulate the specific employment of UAVs with computer software. UAV motion lends itself to simulation, since UAVs are unmanned and their movement can be programmed and predetermined. Simulation also provides an affordable way to test different scenarios without actually flying the UAVs and mobilizing multiple assets.

Simulation is also valuable in that multiple iterations of the same scenario may be executed over and over to gain insight into expected results and variability. Another benefit is that randomness may be introduced into the simulation in order to provide stochastic results over multiple executions, typically referred to as Monte Carlo runs.

[5] Alan Washburn. "Al Washburn's Personal Homepage." 13 January 2003. http://diana.gl.nps.navy.mil/~washburn/.

Since targets do not always move the same way in real life, randomizing their movements is desirable.

To gain insights into the tactics and numbers of UAVs to use, the group ran multiple Monte Carlo simulations of similar scenarios with UAVs searching for enemy surface contacts. The motion of these targets changed randomly on each execution of the simulation. The group varied UAV tactics and numbers over fixed areas. Initial simulation runs explored the use of a single UAV. The tactical flight path (pattern) of this UAV varied in order to determine the best pattern for UAVs among four basic flight patterns evaluated. Using the best pattern for a single UAV, the experiments then investigated the performance of two UAVs in order to determine the optimum flight pattern for multiple UAVs. Using this flight pattern, the study analyzed three types of formations for multiple UAVs, each using the pattern obtained from the two-UAV evaluation. Finally, the team analyzed scenarios with up to 10 UAVs to further explore the effects of increased numbers of UAVs. Each pattern and formation is described in Section 3.1.5.

3.1 Model

This analysis explores focused area cooperative search and identification, a specific portion of the Integrated Project scenario. The scope enabled consideration of UAV search tactics in a region comparable to the vital area. This complements the broad area surveillance problem analyzed at the systems engineering level. Analysts used the Autonomous Littoral Warfare Systems Evaluator (ALWSE) simulation software to model specific area search scenarios. This involved employing UAVs against small enemy surface contacts in multiple scenarios. The tactics and number of UAVs varied, while all other scenario aspects remained constant.

ALWSE is a "simulation toolkit used to develop unmanned systems which operate in the littoral environment."[6] Users create scenarios that define vehicle motion characteristics, navigation errors, sensor characteristics, and control. The software also enables definition of the scenario area, the characteristics and numbers of threats in the area, and termination conditions. It provides for multiple Monte Carlo runs to be

[6] George Gillman, NSWC-PC, Code A84, "ALWSE Introduction Presentation," April 2004.

performed and allows for data analysis. The software is readily adaptable to simulate UAV motion in the surface search role. This was possible by setting the characteristics of the search platforms to resemble those of tactical UAVs within the architecture under consideration by the IPT, and also by defining target characteristics to resemble those of contacts of interest in the tactical scenario.

3.1.1 Assets

For the ease of handling a large system of UVs, the IPT generalizes UAVs into three categories of Large, Medium, and Small. Due to the high performance variation within UAV categories, this characterization helps identify useful UAV attributes and does not specifically single out current and proposed types of UAVs that have a multitude of configurations and capabilities. A summary of the IPT's general definitions for UAVs is in Table 2.

UAV Category	UAV Characteristics
Large	High Endurance, Large Coverage Area, Primary Search and Location Platform, capable of surface search over water and land. Assumed to carry high-powered/large-sized instruments (i.e., Surface Search Radar, Synthetic Aperture Radar (SAR), Inverse Synthetic Aperture Radar (ISAR)).
Medium	Medium Endurance, Medium Coverage Area, Primary ID, Surveillance and Strike Platform (or both). Assumed to carry medium-sized instruments (i.e., small radar < 75 NM, FLIR, missiles or bombs).
Small	Low Endurance, Small Coverage Area, Primary Surveillance/ Targeting. Assumed to carry small-sized instruments (i.e., EO/IR sensors, laser designator).

Table 2. General UAV Definitions.

The IPT also defines the characteristics in Table 3 for ease of modeling and to account for technological capabilities in the 2020 time frame.

Characteristics	Effect	Performance Parameter
Endurance	Loiter Time, Mission Time	High Endurance: ≤ 50 hours Medium Endurance: ≤ 25 hours Low Endurance: ≤ 5 hours
Size	Payload	Large: >2,000 lbs Medium: 100-2,000 lbs Small: < 100 lbs
Function	Flexibility	Single-Function UAV: Search, tracking, ID Platform. Example: Global Hawk (SS, SAR) or Predator (FLIR) OR Generally strike platform (Air to Ground) Example: Unmanned Combat Aerial Vehicle (UCAV) with multiple weapons (missiles and bombs). Multifunction UAV: Surveillance and Strike Capabilities. Example: Predator with small search radar and two missiles. Due to payload capability and altitude requirements for different missions, medium UAVs will be used as multifunction platforms. Their size (large enough to carry multiple sensors or weapons) and operating altitude (low enough to target and release weapons effectively) are ideal for search, location, ID, surveillance, and strike missions.

Table 3. General UAV Characteristics.

Taking these specifications into consideration while the Systems Engineering Team members of the IPT centered their analysis on the large, high endurance UAV, the OR Group analyzed medium UAVs equipped with one sensor. The decision to investigate only medium UAVs simplified the problem and also made tactical sense. Large UAVs are more expensive and potentially less likely to be available, while small UAVs are primarily used for surveillance or targeting, and it is unlikely that they will be used in the SSC mission. For this study, the OR Group created medium UAVs in the ALWSE simulation. The objects' characteristics are summarized in Table 4:

Model UAV Characteristics	
Speed	90 knots
Turn Rate	15 degrees/second
Sensors	FLIR (2.5 NM range) or EO/IR (1.0 NM range)
Endurance	≤ 25 hours
Payload	100-2,000 lbs

Table 4. Model UAV Characteristics.

3.1.2 Targets

Given the Integrated Project's tactical scenario and the expected employment of UAVs in the surface search mission, the model simulates 30 small, low-observable, surface contacts of interest (COIs). Each of these contacts is 30 feet long, maintains a speed of 10 knots, and turns randomly every 15 to 3,000 seconds with a 30-yard turning radius. These targets also correspond to the targets considered in Chapter III of this study (sensor fusion modeling). This study assumes that these threats are below the detection threshold of the larger systems engineering model.

3.1.3 Area

The search model initially focused on a scenario area of 10 NM x 20 NM, considered to be the size of the vital area for a high-value unit. That is, it is an area in which commanders desire the greatest confidence in detection and identification of all targets prior to or upon arrival, within a reasonable amount of time (several hours or less). Low observable targets make this a challenging problem. Later analysis expanded this area to 20 NM x 40 NM to gain insight into tactics with larger numbers of UAVs.

3.1.4 Sensor Range

The study considered two UAV sensors, a FLIR type of sensor with a range of 2.5 NM, and an EO/IR sensor with a range of 1.0 NM. Although larger IR sensor ranges are possible (the Integrated Project study considers IR ranges up to 26 NM using physics-based equations involving line of sight and altitude) much smaller sensor ranges are more practical in a vital area search model.

One justification for these smaller ranges is that FLIR or EO/IR sensor range is highly dependent on field of view. In order to see this relationship, consider that an average display monitor consists of 1,024 pixels on the horizontal axis. Then, calculate the possible view section width at various slant ranges by multiplying the slant range by the sine of the sensor's field of view. Dividing this possible view section width by the average number of pixels (1,024) yields horizontal range resolution in yards per pixel. Comparing these horizontal range resolutions to the National Image Interpretability Rating Standards (NIIRS) of 2 yds/pixel for detection, 0.5 yds/pixel for classification,

and 0.25 yds/pixel for identification results in an array of resolution ranges for various fields of view, as depicted in Table 5.[7]

Field of View (Degrees)	**Slant Range (NM)**		
	Detect	**Classify**	**Identify**
1.5	30	10	5
5	12	3	2
15	4	1	0.5
22.5	3	0.6	0.3
30	2	0.5	0.25
45	1.5	0.3	0.15
60	1.2	0.25	0.1

Table 5. Range requirements for various FOVs.

Because common FLIR and EO/IR sensors have field of views in the 20-60 degree range, the detection ranges of 2.5 NM and 1.0 NM make sense as upper bounds on a sufficiently sensitive sensor. These smaller ranges are also more likely estimates due to variations in the earth's atmosphere, temperature, condensation, cloud cover, thermal-crossover, target size, and sensor resolution.

3.1.5 Flight Patterns

This study evaluated three different types of flight patterns including an offset pattern, alternating pattern, and diminishing squares pattern. These patterns were also evaluated against a random search pattern. In Search and Detection, Washburn states, "given the assumption that the target is stationary and equally likely to be anywhere, all methods are equivalent. However, it can be said that the raster scan method (the alternating or offset patterns in this study) involves the simplest navigation . . . and that the spiral-in method is capable of trapping a slowly moving target."[8]

This study examines these patterns with multiple, nonstationary targets. The expectation is that these differences cause significant differences in the results with each pattern. If the relative speed between the targets and the UAV is small enough, any thorough search pattern is effectively the same as a random search.[9] The UAV and target

[7] See Appendix III.A for complete calculations.
[8] Alan Washburn. Search and Detection, 4th Ed. Institute for Operations Research and the Management Sciences, 2002. pp. 1-2.
[9] Ibid, pp. 2-6.

speed are based on real platforms still expected to perform better employing patterns rather than random searches. A short description of each pattern follows.

3.1.5.1 Offset Pattern

The offset flight pattern, also referred to as a “lawnmower with offset pattern,” features the UAV sweeping back and forth across the search area to detect targets. The UAVs motion is similar to a lawnmower as it moves back and forth through swaths of the vital area. The distance between swaths in this area is equivalent to twice the sweepwidth of the UAV. The sweepwidth of the UAV is equal to twice the range of the UAV’s sensor, as depicted in Figure 13.

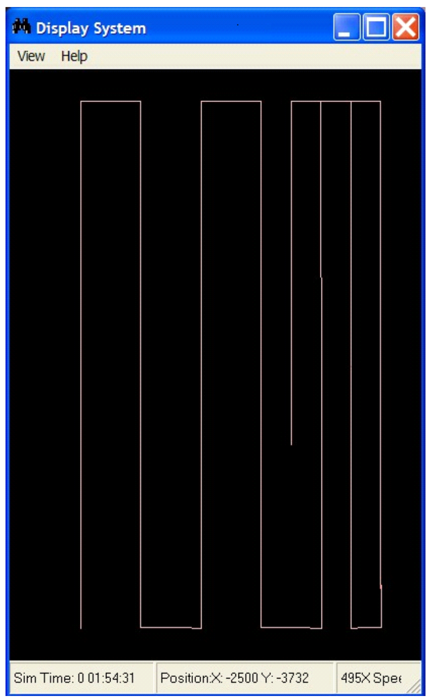

Figure 13. Offset Pattern.

This pattern is called “offset” because once the UAV completes its motion through the search area, it reverses the pattern and sweeps back across the area in a reciprocal fashion, with an offset on the first distance between swaths. This offset is equal to one-half of the sweepwidth in an effort to prevent the UAV from traversing the same exact path on its way back across the search area. The UAV simply continues this

pattern back and forth across the search area, again reciprocating the offset pattern each time it completes its sweep of the area.

3.1.5.2 Alternate Pattern

The alternate flight pattern is the same as the lawnmower search pattern described above, with one exception. After it completes its first sweep of the area, the UAV rotates its swaths 90 degrees and searches the area in a perpendicular direction (i.e., cuts horizontally instead of vertically). This is sometimes called a "waffle pattern" as the UAV crisscrosses throughout the space as shown in Figure 14. Upon completion of its second scan through the area, the UAV simply rotates another 90 degrees and the pattern starts all over again.

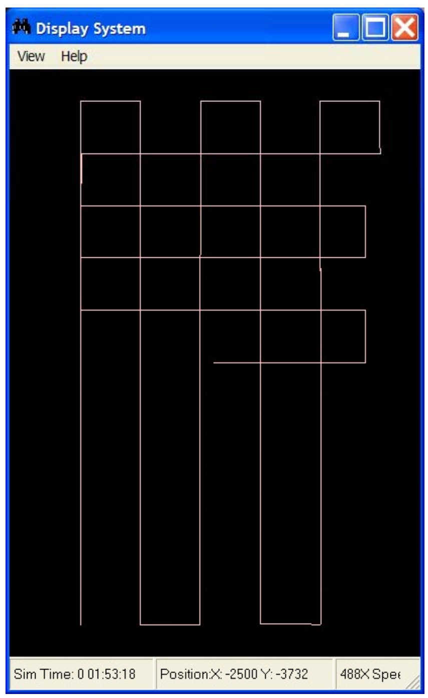

Figure 14. Alternate Pattern.

The offset pattern might be chosen over the alternate pattern if it was suspected that targets were more likely to move perpendicularly to the offset pattern described earlier, in such a manner that they might be trapped earlier by the alternate pattern.

3.1.5.3 Diminishing Square Pattern

The diminishing squares flight pattern follows along the perimeter of the vital area in a decreasing square pattern until it sweeps the entire search area. Each leg or side of the diminishing square is spaced one sweepwidth apart so that there is no overlap in search area as shown in Figure 15.

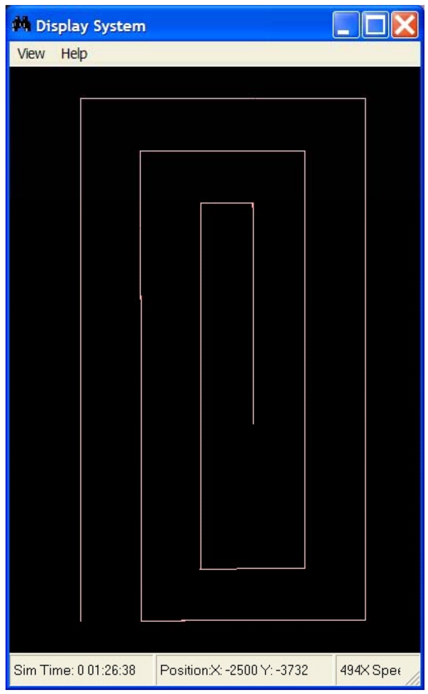

Figure 15. Diminishing Square Pattern.

Upon completion of the pattern (when the UAV reaches the center of the search area) the UAV moves back to the original starting position and resumes the same pattern.

3.1.6 Tactics Involving Multiple UAVs

This study also looked at ways to employ multiple UAVs to complete the surface search mission over a given area. The purpose of this part of the study is to evaluate tactics by multiple UAVs, including multi-UAV formation, sector, and converging tactics.

3.1.6.1 Formation Tactic

The first option for consideration is a multiaircraft formation tactic. For this option, multiple UAVs move throughout the search area using the same flight pattern, while spaced equidistantly. The space between the UAVs is equal to two times the range of the UAV's sensor. In general, the effect of the formation pattern is to increase the sensor area or lateral sweepwidth. The UAVs emulate swarming technology as they work together to cover the vital area faster. The downside to this tactic is that formation flight is inherently dangerous and requires a high degree of coordination between aircraft and airspace control measures to prevent midair collision.

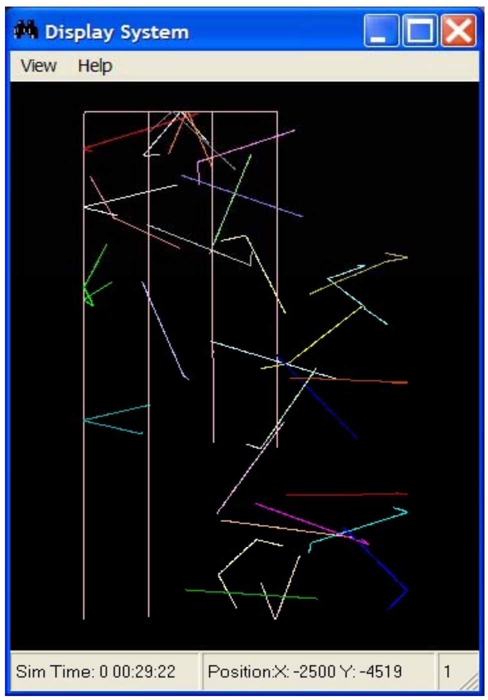

Figure 16: Formation Tactic.

3.1.6.2 Sector Tactic

The second option is the sector tactic. For this option, the search area is divided by the number of UAVs and each UAV searches in a separate sector from the others.

The sectors for this tactic may either be on the vertical axis or the horizontal axis of the search area—both are explored during this study. The benefits of this "divide and conquer" technique include lateral airspace separation to decrease the potential for

collisions. However, this tactic still requires a high degree of coordination between assets. The potential for collisions along common sector boundaries may require altitude separation as well.

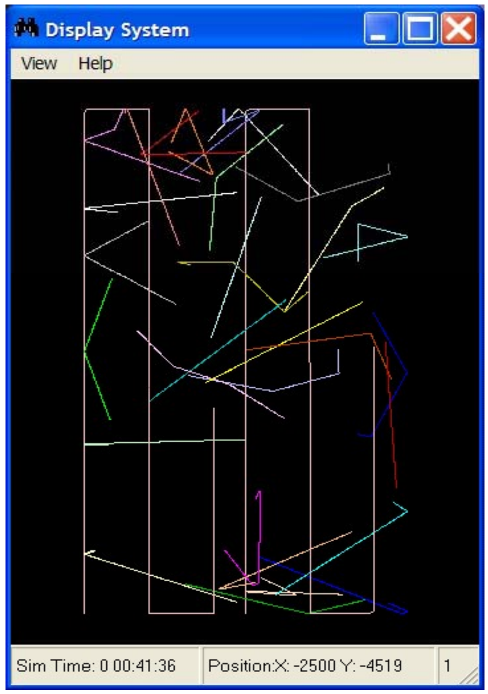

Figure 17. Sector Tactic.

3.1.6.3 Converging Tactic

Finally, two UAVs can start at opposite ends of the search area and work their way towards the center in the converging tactic. This tactic may be appropriate if it is suspected that targets are fleeing to the sides of the search area. This tactics also requires a high degree of coordination between assets, as well as constant monitoring to ensure airspace separation once the UAVs get closer to each other in the center of the vital area.

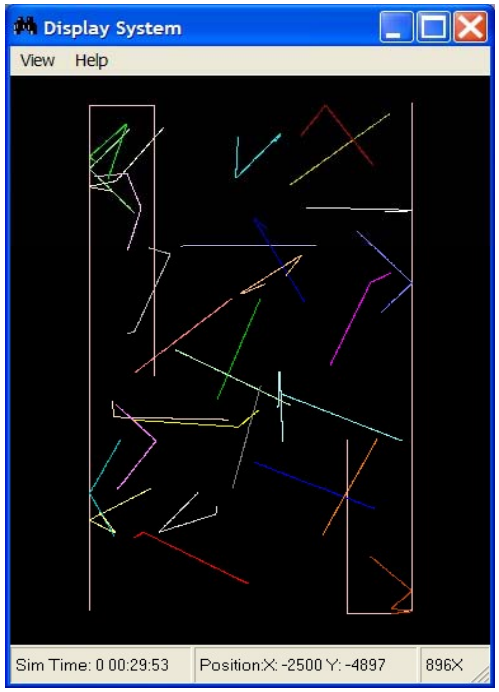

Figure 18. Converging Tactic.

3.2 Measures of Effectiveness

The measure of effectiveness in each scenario is the probability of detection over coverage ratio. Coverage ratio is determined by:

$$\text{Coverage ratio} = \frac{VWt}{A}.$$

V is the velocity of the UAV, *W* is the sweepwidth, *t* is time, and *A* is the vital search area size. Time is measured as each of the 30 targets in the vital area is detected, yielding the proportion of detections over time.

The probability of detection is calculated by:

$$p(D) = \frac{\text{Number of targets detected}}{\text{Total number of targets}}.$$

In this way, detection may be compared as a function of coverage ratio and may be estimated in a spreadsheet.

Over the course of many runs of each scenario, experiments generated the proportion of contacts detected as a function of coverage. For a given coverage ratio, the

team then calculated the sample mean proportion of contacts detected. For a number of runs n sufficiently large, p, the probability of detection, is distributed according to: $p \sim N(\overline{p}, \frac{s_p}{\sqrt{n}})$, where $\overline{p}$ is the average probability of detection and s_p is the sample standard deviation of proportion of detections at the coverage ratio computed for a specific time duration. The OR Group selected a 95% confidence interval on the probability of detection using a margin of error equal to approximately $2\frac{s_p}{\sqrt{n}}$.

In order to determine the minimum number of Monte Carlo runs required in order to achieve significant results with this confidence interval, analysts performed 10, 30, 50, and 100 runs of the scenario with a single UAV flying the alternate pattern against 30 targets. As the number of runs, n, increased, the mean time to achieve 100% probability of detection and 95% confidence intervals did not change significantly and seemed to converge past n = 50, making it sufficiently large. Figure 19 shows the differences in time to reach 100% p(Detection) for this analysis. Note that the improvement (indicated by the slope) with 100 runs is not significantly different than for 50 runs. Therefore, 50 runs is sufficient.

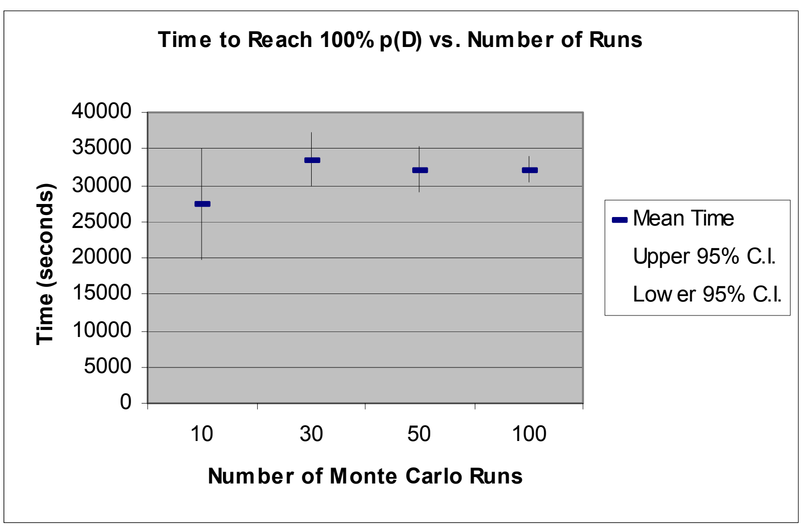

Figure 19. Time to Reach 100% P(D) vs. Number of Runs.

An exception to this experimental design occurred during the analysis of random search and detection due to a limitation of the ALWSE package. For this scenario, time

requirements limited analysis to only 10 Monte Carlo runs because each run had to be performed individually. The inability to batch process significantly increased the amount of time required to complete all runs. However, the confidence intervals on these 10 runs still converge quickly enough to provide interesting results, making this design structure sufficient.

4. RESULTS AND ANALYSIS

As mentioned in Chapter III, Section 3, in order to gain insights into the tactics and numbers of UAVs to use, the OR Group ran multiple Monte Carlo simulations of similar scenarios with UAVs searching for enemy surface contacts. The motion of these targets occurred randomly on each execution of the simulation. The analysis explored performance by varying UAV tactics and numbers of UAVs over fixed areas.

At first, the analysis considered patrols by a single UAV in order to determine the best pattern for UAVs among the three basic flight patterns. Then, using the best pattern for a single UAV, the same experiment was conducted for two UAVs in order to determine the optimum flight pattern for multiple aircraft. Using this flight pattern, the OR Group then analyzed the three types of formations (converging, sector, formation) for multiple UAVs. Finally, the group analyzed scenarios with up to 10 UAVs to further explore the effects of increased numbers of UAVs. This process is depicted in Table 6.

After the completion of multiple simulation runs for each scenario, graphs captured the Probability of Detection versus Coverage Ratio, as summarized in Table 6, with references to the applicable graph for each scenario.

Goal	Types of Runs	Grid (NM)	Friendly (UAVs)	Sensor	Results
To explore different patterns	Offset	10_X_20	1	FLIR	Figure 2.8
	Alternate	10_X_20	1	FLIR	Figure 2.8
	Diminishing Squares	10_X_20	1	FLIR	Figure 2.8
	Offset	10_X_20	1	EO/IR	Appendix 2B
	Alternate	10_X_20	1	EO/IR	Appendix 2B
	Diminishing Squares	10_X_20	1	EO/IR	Appendix 2B
	Random	10_X_20	1	FLIR	Appendix 2C
	Random	10_X_20	1	EO/IR	Figure 2.9
To explore different tactics	Formation-Offset	10_X_20	2	EO/IR	Appendix 2D
	Formation-Alternate	10_X_20	2	EO/IR	Appendix 2D
	Formation-Diminishing	10_X_20	2	EO/IR	Appendix 2D
	Formation-Alternate	10_X_20	2	EO/IR	Appendix 2F
	Formation-Alternate	10_X_20	3	EO/IR	Appendix 2F
	Formation-Alternate	10_X_20	4	EO/IR	Appendix 2F
	Formation-Alternate	10_X_20	5	EO/IR	Appendix 2F
	Converging	10_X_20	2	EO/IR	Appendix 2E
	Sector-Vertical Split	10_X_20	2	EO/IR	Appendix 2E
	Sector-Horizontal Split	10_X_20	2	EO/IR	Appendix 2E
	Sector-Vertical Split	10_X_20	3	EO/IR	Appendix 2E
	Sector-Horizontal Split	10_X_20	3	EO/IR	Appendix 2E
	Sector-Vertical Split	10_X_20	4	EO/IR	Appendix 2E
	Sector-Horizontal Split	10_X_20	4	EO/IR	Appendix 2E
	Sector-Vertical Split	10_X_20	5	EO/IR	Appendix 2E
	Sector-Horizontal Split	10_X_20	5	EO/IR	Appendix 2E
To explore different numbers of UAVs	Sector-Vertical Split	10_X_20	2	EO/IR	Appendix 2G
	Sector-Vertical Split	10_X_20	3	EO/IR	Appendix 2G
	Sector-Vertical Split	10_X_20	4	EO/IR	Appendix 2G
	Sector-Vertical Split	10_X_20	5	EO/IR	Appendix 2G
	Sector-Vertical Split	10_X_20	6	EO/IR	Appendix 2G
	Sector-Vertical Split	10_X_20	7	EO/IR	Appendix 2G
	Sector-Vertical Split	10_X_20	8	EO/IR	Appendix 2G
	Sector-Vertical Split	10_X_20	9	EO/IR	Appendix 2G
	Sector-Vertical Split	10_X_20	10	EO/IR	Appendix 2G

Table 6. Scenario Description.

Figure 20 shows that when comparing patterns for a single UAV with FLIR sensor, the alternating pattern is clearly more effective. Dotted lines identify the upper and lower bounds of the 95% confidence interval. Offset and alternate patterns are significantly better than diminishing squares above 0.35 coverage ratio, and alternate is significantly better than offset above 0.85 coverage ratio.

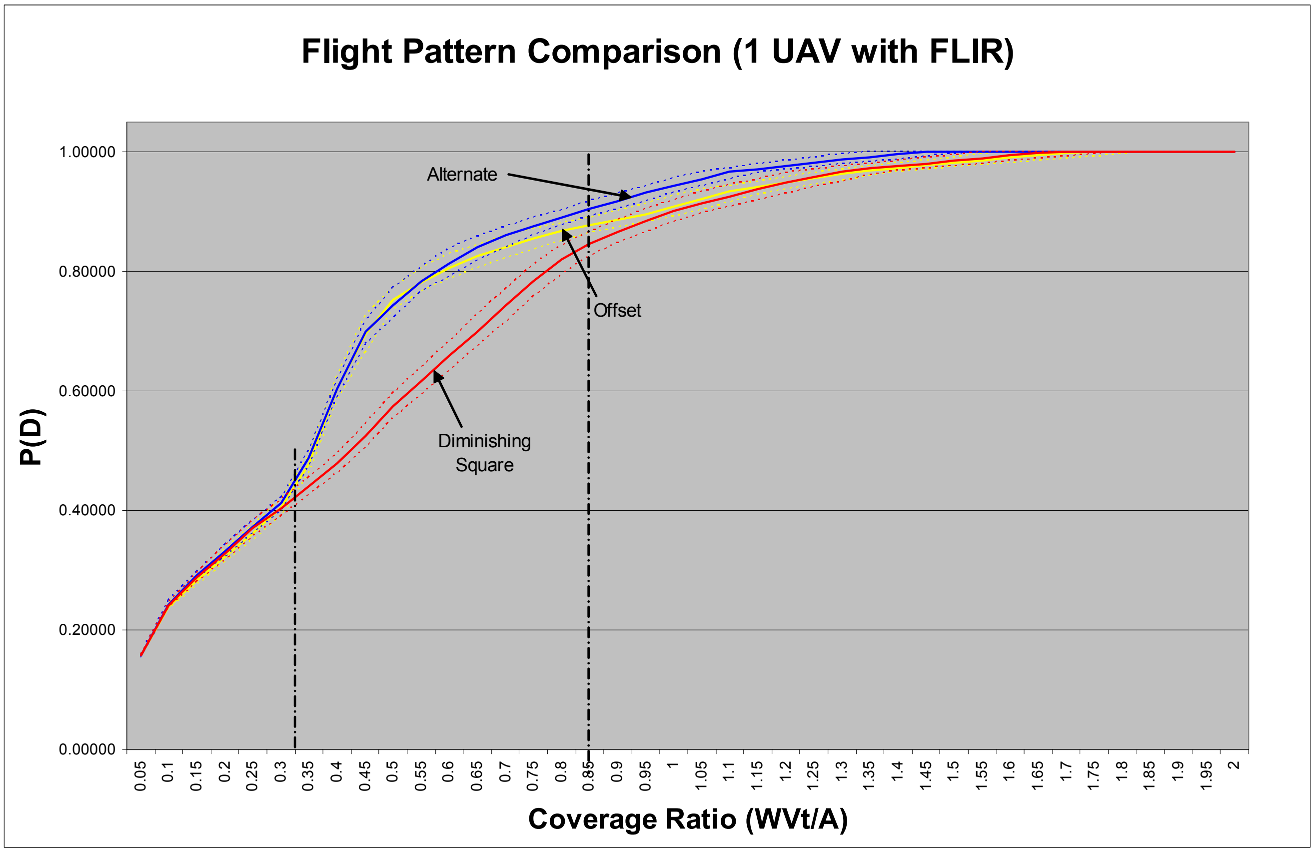

Figure 20. Flight Pattern Comparison (1 UAV with FLIR).

Overall, the alternating pattern proved superior when compared against an offset or a diminishing squares profile. Figure 21 shows the performance of the alternate being significantly better than the random search pattern.

The theoretical random probability of detection for a random search is also plotted in Figure 21. This curve is plotted from the equation P(D)=1-exp((-1*WVt)/A).[10] The experimental results for the alternating search closely adhere to the theoretical random search values. Section 3.1.5 explained that this might occur when the relative speeds

[10] Alan R. Wasburn. Search and Detection ,4th Ed. Institute for Operations Research and the Management Sciences, pp. 2-3, 2003.

between the searcher and the evader are smaller. However, this study used a 9:1 speed ratio (searcher to evader); when comparing the ALWSE random search with the alternate pattern, the alternate pattern was significantly better. The difference in the theoretical random and the ALWSE random search does not invalidate the ALWSE model. The difference is due to the different measures of effectiveness used in the two different random techniques.

In the closed form equation, the assumption is that there is only one contact and it is being repeatedly detected as time (t) goes to infinity. In this experimental study, the ALWSE model used 30 mobile contacts. With each corresponding detection, there is an additional $1/30^{th}$ probability of detection. Hence, P(D) = 100% when all 30 contacts were found. It is also important to note that although the values are different, the shapes of the curves are similar (i.e., they are both exponential). This indicates significant similarity in the underlying characteristics captured in experimentation with those suggested analytically. Similar results were found when the sensor was changed from EO/IR to FLIR (see Appendix III.C).

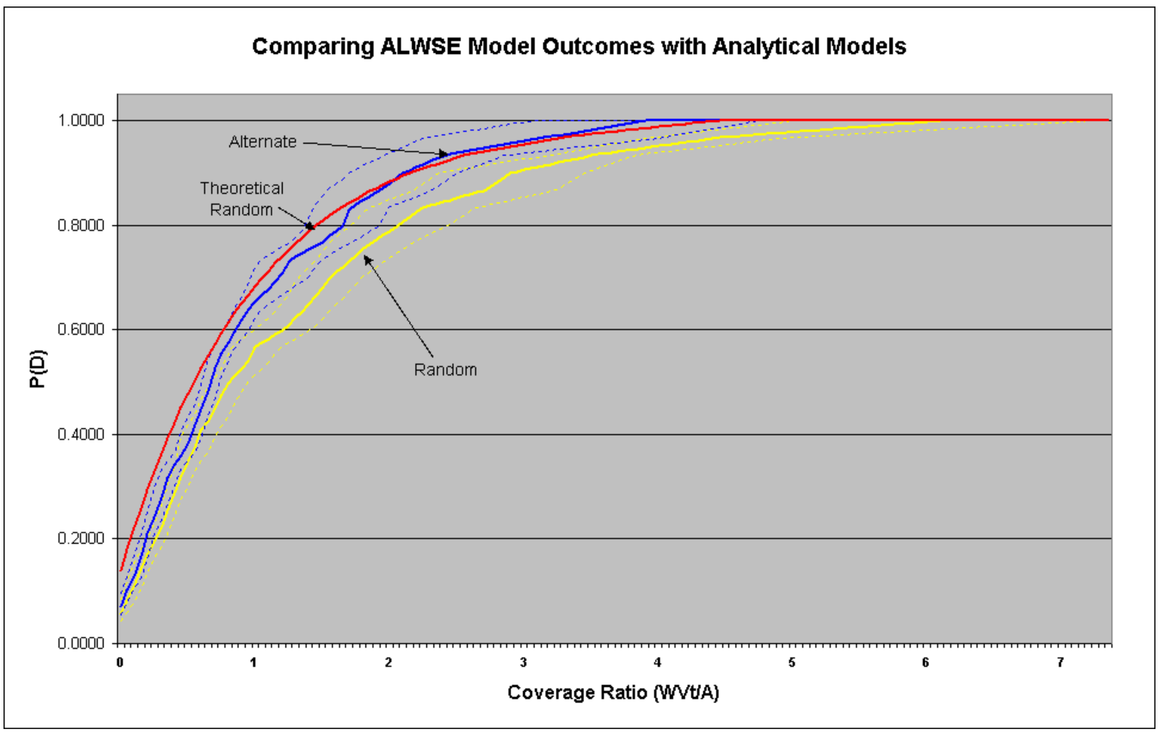

Figure 21. Comparison Single UAV random and lawnmower alternating patterns.

5. CONCLUSIONS

After running a sufficiently large number of simulation experiments, several insights regarding UAV tactical employment emerged. This set of limited area search and detection simulation experiments yielded the following results:

- UAVs (with sensor ranges between 1.0 NM and 2.5 NM) using an alternate pattern outperform those using an offset, diminishing squares or random search pattern. Tactics matter. (Figure 19 and Appendix III.C)
- Use of more than one UAV significantly increases the probability of detection. (Appendix III.G)
- Sector employment within a 10 NM x 20 NM search area is generally more effective than a formation pattern. While tactics matter, it is inconclusive whether formation or sector is better in general. (Appendix III.F)
- Addition of a second UAV to a 10 NM x 20 NM area is estimated to increase search performance by a factor of four. (Appendix III.H)
- Effectiveness also increases with the addition of a third, fourth, and fifth UAV; however, this increase in effectiveness is not as great as the difference between one and two UAVs. (Appendices III.H)
- Beyond five UAVs, in the 10 NM x 20 NM scenario studied, no significant increase in effectiveness is noted. (Appendix III.G)

This study provides quantitative results for systems analysts to determine the relative effectiveness of investments in more UAVs. Clearly, the number of UAVs and tactics they employ directly influence force effectiveness in focused area search.

6. POTENTIAL SOURCES OF ERROR

While the methodology attempted to negate sources of error for this study, modeling did induce some undesired variability. The simulation presented targets that started at the same locations throughout the majority of runs. It was not possible to conduct all runs and vary the target position on each run due to software programming problems associated with each Monte Carlo run. Analysts compensated for this by changing target positions over several sets of runs, and comparing them against each

other to investigate the influence of target position. This did have some effect. For example, in the scenario that compared three UAVs in formation against sectoring the area, differences emerged. Analysts used the best scenario for comparison purposes.

A second source of error involved UAVs and formation flight. UAVs would most likely fly as planes in formation fly—along a diagonal, with vertical and horizontal offset. This type of formation requires the UAV on the outside of turns to move more quickly, and the UAV on the inside of turns to move more slowly. The area searched would therefore be affected. The simulation does not account for these effects. UAVs simply move laterally across each other as they maintain their horizontal separation. This may induce some marginal effects.

With regard to tactics, multiple UAV flight would also involve a certain altitude separation for safety reasons. This altitude variation would influence sensor range that was not accounted for in this study. UAV motion would also be heavily dependent on environmental effects such as wind, rain, and air density. These effects would change platform speed, or sensor performance. While this would serve to modify UAV performance characteristics as a function of time, use of coverage ratio as the independent variable serves to mitigate this shortcoming.

7. FUTURE RESEARCH

This study raises several questions appropriate for future research. While this study provides insight into multiple UAV operations, it assumes that a detailed surface picture is not already available. Cooperative search and identification might involve a high altitude, broad area search UAV, which could cue smaller UAVs on scene for identification. The potential effects on time and efficiency of this search and identification process can also be evaluated. Cooperative use of multiple UAVs to perform concurrent search and identification is another framework for comparison. An interesting study would look at the amount of time required for more efficient UAVs to both investigate an area and loiter on top of any targets of interest. The efficiency and cost of this approach could then be compared against the approach described in this study and the cued search and identification architecture.

IV. SENSOR DATA FUSION MODEL

1. INTRODUCTION AND SUMMARY

The third analysis looks at the Recognized Maritime Picture (RMP) data fusion problem. While there are many aspects to this problem, this study considered the advantages and shortcomings of using several sensors versus only one. The model uses the probability of detection (Pd), the probability of a false alarm (Pf), and the sensor fusion protocol as inputs. To complement the localized search and detection model, the scenario of interest puts sensors against low-observable targets in a cluttered environment. The models use influence diagrams to simulate and check different sensor qualities under this scenario. This simple model can assist in a trade study between sensor quality and shared contact processing.

2. RECOGNIZED MARITIME PICTURE (RMP)

One of the basic elements in combat is intelligence. Part of the intelligence is information about enemy unit locations as well as friendly unit locations. The RMP forms a database of where the units are and whether they are friendly, neutral or hostile, as well as other characteristics like heading, speed, and type. This processed database contains information about other units in the battle space.

Because multiple sensors feed the RMP, this analysis looks at the sensor and data fusion process. The point of comparison is whether greater numbers of lower quality sensors can outperform fewer, high quality ones. While the question of a single sensor against a target is straightforward, this study analyzes the sensor's fusion, and compares the capabilities of two or more sensors compared to a single one.

3. PROBABILITY OF DETECTION AND FALSE ALARM RATE

A sensor should detect targets with the highest probability possible. Likewise, it should not indicate a target when there isn't one; that is, it should have the lowest probability of creating a false alarm, also referred to as false alarm rate. These two probabilities, detection and false alarm, are the most important operational attributes of a sensor. Therefore, the Operations Research (OR) Group looked at these attributes as the criteria for the quality of a system. Other attributes such as range, time of day, and weather dependencies are considered as factors that affect these probabilities.

For a given sensor, Pd and Pf are highly correlated. Engineers can improve a sensor's probability of detection and increase the false alarm rate by decreasing the threshold that discriminates signal from noise. A sensor can exhibit several levels of certainty at different thresholds. Each level has a specified probability of detection and an appropriate false alarm rate. For example, a sensor can have two thresholds for detection. When the first threshold is crossed, the sensor indicates that there may be something there, and when the second threshold is crossed, the sensor will indicate that there is an even higher certainty of a target.

As mentioned, the higher the probability of detection and the lower the probability of a false alarm are, the better the sensor. However, since these two probabilities go up and down together when the threshold is changed, there is no optimal threshold, but rather different values, with appropriate probabilities of detection and false alarm associated at each setting. For example, a radar receives a signal return from a target, but it might be very weak. Establishing a threshold for the power of this return determines whether the sensor can declare it as a target if the power is higher than the threshold. Designers can increase the probability to detect a target by lowering this threshold, but then other noise might also pass the low threshold and be considered as targets. This is a false alarm.

To choose the best threshold, one should take into account the probabilities of a real target appearing, and the costs for a false alarm and for nondetection. For example:

- The higher the threat is to a sector, the higher the probability of a target appearing.
- A cost for a false alarm can be the cost of sending out investigating aircraft, or even launching a missile to intercept it.
- The cost for nondetection can be a delay in warning to the attacked force or letting an intruder pass the guarding sensor, which might result in losing an asset.

4. HIGH THREAT AND LOW THREAT SCENARIOS

If a target arrival is a rare event, the cost for a false alarm is big and the cost for nondetection is low. The threshold should be high, so that the probability of false alarm is small, even though the probability of detection might be low as well. This scenario suits a peaceful region, with no critical targets to defend. The current study refers to this as the “low threat scenario.”

On the other hand, if the probability of a target appearing is high, the cost for a false alarm is small and the cost for nondetection is large. The threshold should be low, so that the probability of detection will be high (even though there will be more false alarms). This scenario suits a threatened sector during wartime. This is the “high threat scenario.”

5. THE USER AS A THRESHOLD

It is easier to think of the threshold as being implemented at the hardware and software systems level of the sensor, but it can also be at the user. The user can act immediately upon seeing any indication at all for a target, or alternatively wait and check whether the indications persist and increase certainty. For example, a sonar operator can perceive a faint echo (a “blip”) and immediately alert the force, or wait and listen whether the signal continues to appear in the following scans. An Electro-Optical (EO) sensor user can alert whenever anything changes in the field of view, or only when a discernable image appears hostile. In this paper, this user-threshold is referred to as the alertness of the user.

A radar’s display can be seen in Figure 22. The figure shows a lot of points on the radar’s display. While some of the points are real targets, others are false ones, caused by clutter reflection or random noise. If the radar operator declares a target whenever he sees a new point on the display, there will be many false alarms. So, the operator should look at the points’ attributes such as stability in time, power, size, shape, and location, and decide whether or not to declare the new points as targets. This is an example of a user-threshold.

Figure 22. A radar's display.

After discussing both systems and user thresholds, assume that a given sensor has several such thresholds, each with different probabilities of detection and false alarm (or some relationship between the probability of detection and the false alarm rate). Regardless of where the determination is made, a sensor's contribution to the RMP has a different significance when considered in the context of a system of sensors.

6. SENSOR FUSION ADVANTAGES AND PROTOCOLS

While improvement of the probability of detection for a given sensor is limited due to increasing false alarm rates, as explained earlier, using two or more sensors may increase probabilities of detection while decreasing the false alarm rate. Consider two modes of operation for two sensors:

1) Treating each sensor as a stovepipe sensor ("stand alone").
2) Treating both sensors as part of a system ("sensor system").

The "stand alone" mode causes both the probability of detection and false alarm rate to increase. For the new system, the false alarm rate is $Pf_1 + Pf_2 - (Pf_1)(Pf_2)$. Assuming that the two sensors are conditionally independent of one another, then the new

probability of detection is 1-(1-Pd_1)(1-Pd_2). If the two sensors are 100% dependent, then whenever one sensor detects a target, the other one detects it as well, and the force gains nothing by the added sensor—only a higher false alarm rate. If the sensors are 100% independent, then the probabilities of detection are much higher than that of a single sensor. For example, if the two sensors are independent, each with a probability Pd for detection, then the new probability of detection is $1-(1-Pd)^2$, but the false alarm rate is still higher. So, there is more to gain from independent sensors. In real life there is typically dependence among the sensors; operations planning tries to minimize this dependence by choosing the right sensors, the right locations for the sensors, and spectrum management, among other requirements. A system of sensors requires a protocol to combine the output. Two extreme algorithms are:

1) Both stand alone, treating any signal as a target.
2) "Both or nothing"—a target is declared only if both sensors detect it.

The first protocol increases both the probability of detection and the false alarms rate, while the second algorithm decreases them both. Table 7 summarizes this.

Protocol	Probability of Detection	False Alarm Rate
Both stand alone	+	+
Both or nothing	–	–

Table 7. Different protocols' effects on Pd and Pf.

Different probabilities and operating environments will lead to choosing the appropriate protocol.

While the protocols in Table 7 are very simple, the algorithms may have many other parameters that might improve the system as a whole. The algorithm's parameters may include the following:

1) Using more than two sensors (detection by m out of n sensors).
2) Sensors that provide more kinematics about the target (speed, heading, altitude, etc.).

3) Sensors that give confidence estimation for the certainty level of the detection being a real target.
4) Using previous detections, integrated over time.
5) Directing other sensors to a suspected area.
6) Applying tactical intelligence.
7) Environmental conditions such as weather or clutter.
8) Contrast such as day or night for visual sensors, and hot or cold for IR.

This analysis considers the environment and contrast as influential parameters in assessing the value of a fused sensor system. To better understand the role of sensor data fusion, this study evaluates how these scenarios influence outcomes.

7. RELEVANT SCENARIOS

With a high probability of detection against a target, there is little need for more than one sensor, which can operate using a high threshold and achieve low false alarm rate. However, in our analysis, we consider a target with a low-observable signature. This can be a small, rubber-hulled boat against two radars.

Another interesting scenario is when there are two distinctively different sensors: one sensor with a fast scanning ability, but a low probability of detection, and the other sensor with a slow scan, but high Pd. An example of this can be a search radar and a UAV with an EO sensor, looking for intruding ships similar to the cooperative search and identification problem cited in the second study.

8. ASSUMPTIONS

The main assumption in this model is that the probability of detection is a function of environmental and target contrast parameters, and that it is sensitive to these parameters. When these parameters are held constant, the probability of detection is either very high or very low. That is, autocorrelation between detections is strong. Detection and false alarm probabilities are generated over a time period, not per look, and represent events based on cumulative results.

The objective of this work is to help the Integrated Project Team (IPT) understand the trade-offs of using multiple sensors at the same time in the same area, by considering

various performance characteristics as defined by the probability of detection and the false alarm rate. Each target presentation was assumed independent and that false alarms are related or considered so by the operators. While sensor engineers considered specific target scenarios for individual component performance, this operational model views the overall capabilities for the sensors and analyzes how performance may be enhanced with fusion.

9. SIMPLE MODEL I – COOPERATIVE SEARCH AND IDENTIFICATION

In this model there are two sensors. One can search large areas, but cannot identify the targets; the other sensor has limited area coverage, but with a resolution good enough to identify the targets. For example, consider a combination of a search radar and EO sensor where the radar has a high probability of detection and a high false alarm rate. When the radar detects a contact, it sends the Unmanned Aerial Vehicle (UAV) to investigate. Conversely, the EO sensor onboard a small UAV has a small probability of detection due to the "soda-straw" effect, as well as a small probability of false alarm. When the UAV detects a contact, it can identify it by itself.

This model is time-dependent model, and so we could not use an influence diagram or some other simple tool to simulate it; rather, we would have needed to write a simulation. The analysis in Chapter II considers a focused area search, which provides some insights into complementary search by less capable sensors in limited size areas. The specific coordinated search process simulation is suitable for future research.

10. SIMPLE MODEL II – LOW OBSERVABLE TARGET

Considering low observable targets that might pose a potential danger, detection might be difficult due to a combination of a small contrast and a cluttered environment. As illustrated by Figure 23, "Day-Night" represents contrast and "Weather" represents the clutter in the environment for an EO sensor. The software application GeNIe assists in development of appropriate influence diagrams.

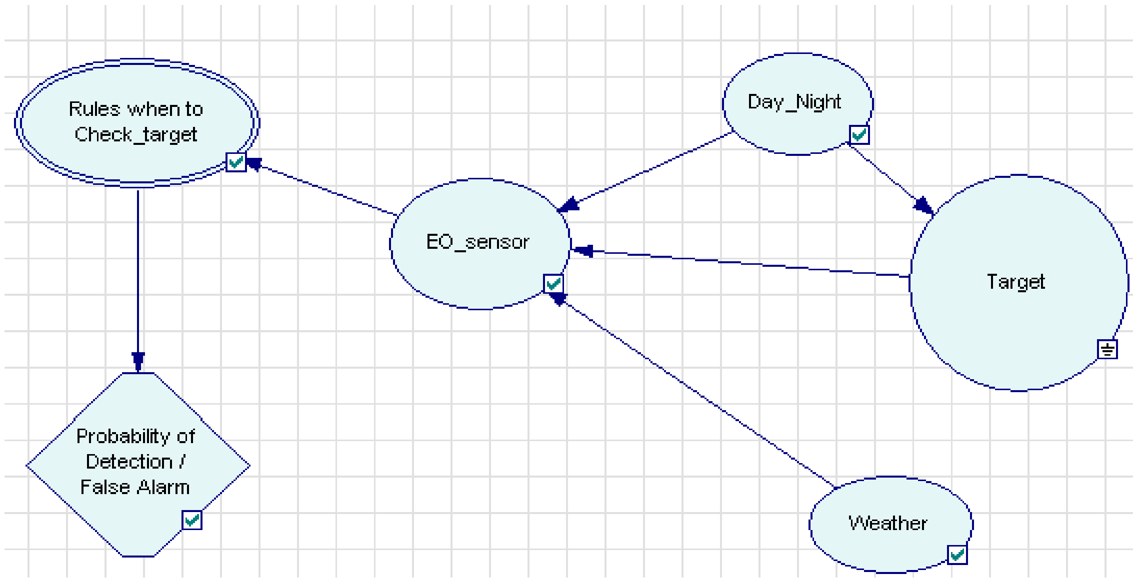

Figure 23. Detection model with a single sensor.

As shown in Figure 23, the parameters that affect the sensor are the weather (good or bad) and the time of day (day or night). The rules are very simple for a single sensor—either "check any potential detection" or "check only certain detections." This becomes more complex with multiple sensors. The EO_sensor object receives as input the time of day, the weather conditions, and whether there is a real target or not. Its output is the probability of detecting a target reflecting three levels of certainty for sensor assessment:

- No target.
- Possible target.
- Certain target.

Each of these outcomes reflects design assessments by the sensor of the level of confidence in the presence of a target.

Absent a real target, there is a nonzero probability that a sensor generates a false alarm. Contrast can also influence the probability that a sensor assumes that there is a target (the arrow from the Day_Night to the Target). For a three-sensors system, the influence diagram looks like Figure 24.

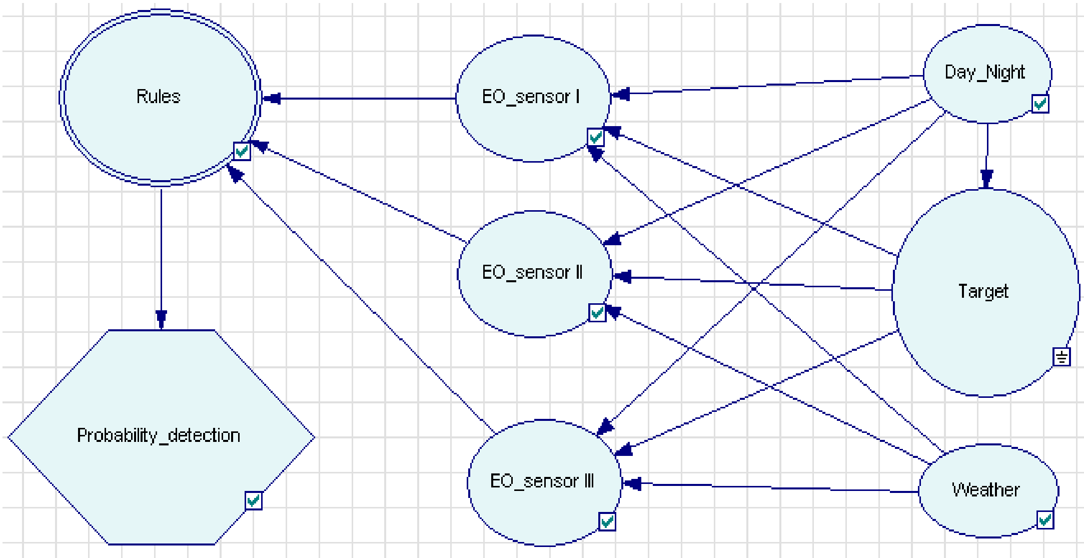

Figure 24. Detection model with three sensors.

While this influence diagram resembles that of the single sensor system, the rules are more complicated. For the single sensor system described earlier, there are only two possible rules:

- Consider any possible detection as a target; or,
- Consider only certain detections as targets.

For a three-sensors system, there are many more rules. Examples for these rules are:

- At least one sensor with certain detection.
- At least one sensor with certain detection and one sensor with a possible detection.
- All sensors must indicate at least a possible detection.

Table 8 summarizes the combinations for all the rules. It also presents computational results for a midgrade sensor. Sensor quality reflects the ability of the sensor to discriminate varying contrast targets in different levels of clutter. Rather than make this study based on specified quality sensors, sensitivity analysis provides an idea of system effectiveness based on various quality sensors.

The study focuses on how integrating protocols enhance or detract from the system's ability to detect targets or be susceptible to false alarms. In Table 8, the rules are explained in abbreviation. The use of "Any" and "Certain" refers to:

- Any - any single detection ("certain" or "possible target") is enough to declare a potential target.
- Certain - a single "certain" detection is enough.

Number of Sensors	#	Rules	P(detection)	P(false alarm)
1	1	Any detection	0.76	0.22
	2	Certain detections only	0.59	0.05
2	3	Any detection	0.92	0.37
	4	Any detection by at least two sensors	0.59	0.07
	5	At least one certain detection	0.8	0.09
	6	At least one certain detection and one possible detection	0.57	0.03
	7	Two certain detections	0.39	0.01
	8	One certain detection or two possible detections	0.82	0.13
3	9	Any detection	0.97	0.48
	10	Any detection by at least two sensors	0.85	0.17
	11	Any detection by three sensors	0.5	0.03
	12	At least one certain detection	0.89	0.13
	13	One certain detection or two possible detections	0.92	0.2
	14	One certain detection or three possible detections	0.89	0.14
	15	At least one certain detection and one possible detection	0.79	0.07
	16	At least one certain detection and two possible detections	0.47	0.02
	17	At least two certain detections	0.63	0.02
	18	At least two certain detections or three possible detections	0.67	0.04
	19	At least two certain detections and one possible detection	0.43	0.01
	20	Three certain detections	0.27	0

Table 8. Rules and results for a midgrade sensor.

In general, using more sensors results in better performance in both maximizing detections and minimizing false alarms. With a fixed number of sensors, system designers can increase one of the probabilities at the expense of the other, improving Pd, but worsening Pf. There are no dominant rules. Some rules do perform better than others in both MOEs, however. For example, requiring two certain detections (rule #17) outperforms a system that accepts any level of detection from all three sensors (rule #11).

Figure 25 illustrates the results of the probability of detection versus the probability of a false alarm for midgrade sensor systems with one, two or three sensors. Each point represents the results for a specific rule, specified in Table 8. The lines between the points serve to group performance of each system. They represent a mixed strategy, meaning that by working part of the time according to one rule, and part of the time according to a different rule, we can choose a location on the line between the two points associated with these rules. Dividing the activity time between the two sensors is one way to achieve a mixed strategy. Another way can be by randomly choosing which rule to follow, each time a rule is needed. For a single sensor system, working 50% of the time according to rule #1, and 50% of the time according to rule #2, yields performance at the center single-sensor line, with Pd = (76%+59%)/2 = 67.5%, and Pf = (22%+5%)/2 = 13.5%.

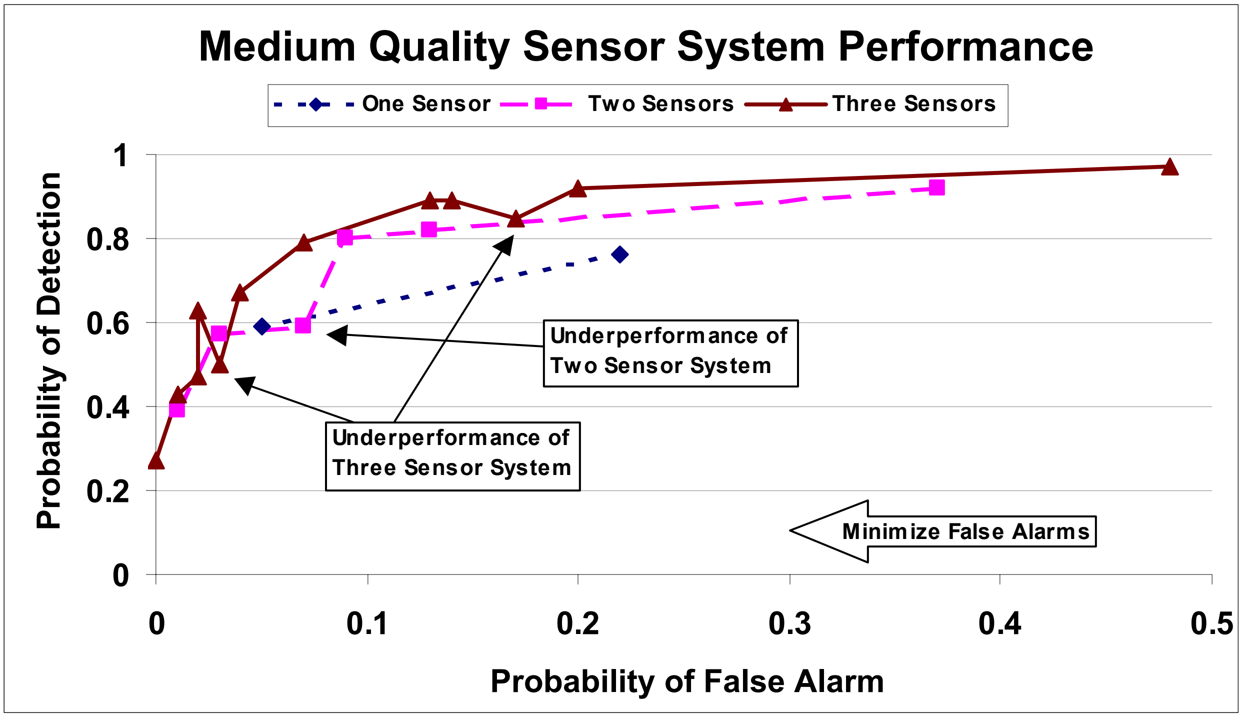

Figure 25. Pd vs. Pf for a midgrade sensor.

The best place to be on the graph is the upper left corner, corresponding to a high Pd and a low Pf. The performance frontier moves further in the direction of both objectives as the number of sensors increases. The curves represent the trade-space for systems designers. Specifying the acceptable lower limit for the probability of detection and upper limit for the probability of false alarm, system engineers can employ a linear combination among appropriate rules, using the minimal number of sensors that comply with these limits, supports cost effectiveness, but not necessarily survivability. For this midgrade sensor, there are rules that will always be rejected since others exhibit better performance measures.

11. SENSITIVITY ANALYSIS

To determine the system's sensitivity to sensor quality, the model incorporated two other sensors, one of which is better and one of which is worse. Compared to the midgrade sensor previously analyzed, the better sensor has equal or higher probabilities of detection and equal or lower probabilities of false alarm in various operating environments. The lower quality sensor has equal or worse performance in these

respects; refer to Appendix IV.A for the parameters of these sensors. The results are in Appendix IV.B.

The changes in sensor quality yield the following outcomes:

- As expected, the better the sensor the better the results for a multiple sensor system. This validates the basic construct of this model.
- For a specific sensor, some rules are inferior to others in both probabilities of detection and false alarm. However, the same rules might not be inferior for a different sensor. The different relations between the certainty levels of the sensors are the reason for this anomaly. For example, it may happen that for a sensor the "possible detection" level is useless due to a very high probability of false alarm at this level. The protocol then uses only the highest confidence detection level. For a different sensor this level may not be enough due to a very low probability of detection, regardless of false alarm rate.

The analysis also compared a single good sensor to a two midgrade sensor system, and to a three low quality sensor system, depicted in Appendix IV.B. In general, the three low quality sensor system is better than the two midgrade sensor system. The results also show that the single good sensor is the best in its range of values—it is inferior to no other system. However, the highest achievable probability of detection, as well as the smallest probability of false alarm, is achieved by the three low quality sensor system. Multiple moderate fused sensors can outperform fewer better sensors.

12. CONCLUSIONS

Using more sensors can improve the probabilities of detection, as well as decreasing probabilities of experiencing false alarms if the correct algorithm is selected. For a fixed number of sensors, most of the time, the improvement in one of these probabilities comes at the expense of the other. In some cases, there are inferior rules, which will produce lower probability of detection, as well as a higher probability of having a false alarm.

As shown, better performing rules are both a function of the thresholds for the probability of detection and probability of false alarm, and of the number and quality of

the sensors. The system trade is to determine the value of detection, quantifying the penalty imposed by false alarm, and establish a rule that fits these thresholds. Even though theoretical estimations for a system's performance can be optimistic compared to the real performance, sensitivity analysis shows better performing systems and protocols in a number of environments.

13. FUTURE WORK

This analysis features a scenario in which the probabilities of detection and false alarm were dependent on contrast and clutter. This analysis can include other attributes of the scenario, such as the observable signature of a target, different sensors, or more levels of detection confidence. The use of an influence diagram can simplify the work, and provide a basis for this trade-off analysis between the quality of the sensors, the quantity, and fusion protocols. However, it is not easy to capture other aspects of the scenario, like temporal and spatial dependencies, in an influence diagram. To simulate these dependencies, as well as other things, we believe that the influence diagram will not be enough, and a more specific physics-based simulation is needed in order to fully account for all of the parameters. As mentioned before, the cooperative search and identification model makes use of a simulation for appropriate analysis.

APPENDIX I – ACRONYM LIST

ALWSE	Autonomous Littoral Warfare Systems Evaluator
AO	Area of Operations
AOI	Area of Interest
AUV	Autonomous Underwater Vehicle
C4I	Command, Control, Communications, Computers, and Intelligence
CCOI	Critical Contact of Interest
COI	Contact of Interest
COP	Common Operational Picture
EO	Electro-Optical
FLIR	Forward-Looking Infra-Red
GAMS	General Algebraic Modeling System
IPT	Integrated Project Team
IR	Infra-Red
ISAR	Inverse Synthetic Aperture Radar
KM	Kilometers
LEO	Low Earth Orbit (Satellite)
MOE	Measure of Effectiveness
MOP	Measure of Performance
NIIRS	National Image Interpretability Rating Standarad
NM	Nautical Miles
OR	Operations Research
Pd	Probability of Detection
Pf	Probability of False Alarm
RADCOM	Radio Communications Node
RF	Radio Frequency
RMP	Recognized Maritime Picture
SAR	Synthetic Aperture Radar
SoS	System of Systems
SSC	Surface Search and Control
TDSI	Temasek Defense Systems Institute
UAV	Unmanned Aerial Vehicle
UCAV	Unmanned Combat Vehicle
USV	Unmanned Surface Vehicle
UUV	Unmanned Underwater Vehicle
UV	Unmanned Vehicle

APPENDIX II

GAMS Code for Baseline Scenario

```
*============================ Generic scenario ONE =============================
*Filename: comms_gs1
*Using importance with just either 0 or 100 and generic grid of 17x25.
*Comms range assumed to be 10km or 5.4NM
*Baseline scenario.

*===================================== SETS =====================================
*(1,1) starts from the top left corner
SETS     I column grid number /1*29/
         J row grid number /1*25/;

*================================== PARAMETERS ================================
*Importance of zones, or can look at it as penalty if not covered
*Input values from 0(least important) to 100(most important),
*intermediate values are allowed, but not used in this version.
PARAMETERS       importance(i,j) /
         1.1=0, 2.1=0, 3.1=0, 4.1=0, 5.1=100, 6.1=100, 7.1=0, 8.1=0, 9.1=100, 10.1=100, 11.1=0, 12.1=100, 13.1=0, 14.1=100, 15.1=0,
16.1=0, 17.1=0,
         2.2=0, 3.2=100, 4.2=0, 5.2=100, 6.2=100, 7.2=0, 8.2=0, 9.2=0, 10.2=100, 11.2=0, 12.2=0, 13.2=0, 14.2=100, 15.2=0, 16.2=0,
17.2=0, 18.2=0,
         2.3=0, 3.3=0, 4.3=0, 5.3=0, 6.3=0, 7.3=0, 8.3=0, 9.3=0, 10.3=0, 11.3=100, 12.3=0, 13.3=100, 14.3=0, 15.3=100, 16.3=0,
17.3=0, 18.3=0,
         3.4=0, 4.4=100, 5.4=0, 6.4=0, 7.4=0, 8.4=0, 9.4=100, 10.4=100, 11.4=0, 12.4=0, 13.4=0, 14.4=0, 15.4=0, 16.4=0, 17.4=0,
18.4=100, 19.4=0,
         3.5=0, 4.5=100, 5.5=0, 6.5=0, 7.5=100, 8.5=0, 9.5=0, 10.5=0, 11.5=0, 12.5=0, 13.5=0, 14.5=0, 15.5=0, 16.5=100, 17.5=0,
18.5=0, 19.5=0,
         4.6=0, 5.6=0, 6.6=0, 7.6=100, 8.6=0, 9.6=0, 10.6=0, 11.6=0, 12.6=0, 13.6=0, 14.6=100, 15.6=0, 16.6=0, 17.6=0, 18.6=0,
19.6=0, 20.6=0,
         4.7=0, 5.7=0, 6.7=0, 7.7=0, 8.7=0, 9.7=0, 10.7=100, 11.7=0, 12.7=0, 13.7=0, 14.7=100, 15.7=100, 16.7=0, 17.7=0, 18.7=0,
19.7=0, 20.7=0,
         5.8=0, 6.8=100, 7.8=100, 8.8=0, 9.8=0, 10.8=0, 11.8=100, 12.8=0, 13.8=0, 14.8=0, 15.8=0, 16.8=0, 17.8=100, 18.8=100,
19.8=0, 20.8=0, 21.8=0,
         5.9=0, 6.9=0, 7.9=0, 8.9=0, 9.9=0, 10.9=0, 11.9=100, 12.9=0, 13.9=0, 14.9=0, 15.9=0, 16.9=0, 17.9=0, 18.9=100, 19.9=0,
20.9=0, 21.9=0,
         6.10=0, 7.10=0, 8.10=100, 9.10=0, 10.10=0, 11.10=0, 12.10=0, 13.10=0, 14.10=100, 15.10=100, 16.10=0, 17.10=0, 18.10=0,
19.10=0, 20.10=0, 21.10=100, 22.10=0,
         6.11=0, 7.11=0, 8.11=100, 9.11=0, 10.11=100, 11.11=0, 12.11=0, 13.11=0, 14.11=0, 15.11=0, 16.11=0, 17.11=0, 18.11=0,
19.11=0, 20.11=0, 21.11=0, 22.11=0,
         7.12=0, 8.12=0, 9.12=0, 10.12=0, 11.12=100, 12.12=0, 13.12=0, 14.12=0, 15.12=0, 16.12=0, 17.12=100, 18.12=0, 19.12=0,
20.12=0, 21.12=0, 22.12=0, 23.12=0,
         7.13=0, 8.13=0, 9.13=0, 10.13=0, 11.13=0, 12.13=100, 13.13=0, 14.13=0, 15.13=100, 16.13=0, 17.13=0, 18.13=0, 19.13=0,
20.13=0, 21.13=0, 22.13=0, 23.13=0,
         8.14=0, 9.14=100, 10.14=0, 11.14=0, 12.14=0, 13.14=0, 14.14=0, 15.14=0, 16.14=100, 17.14=0, 18.14=0, 19.14=0, 20.14=0,
21.14=0, 22.14=0, 23.14=0, 24.14=0,
         8.15=0, 9.15=0, 10.15=0, 11.15=0, 12.15=0, 13.15=0, 14.15=100, 15.15=0, 16.15=0, 17.15=0, 18.15=0, 19.15=0, 20.15=0,
21.15=100, 22.15=0, 23.15=0, 24.15=0,
         9.16=0, 10.16=0, 11.16=0, 12.16=0, 13.16=0, 14.16=0, 15.16=0, 16.16=0, 17.16=0, 18.16=100, 19.16=0, 20.16=0, 21.16=0,
22.16=0, 23.16=0, 24.16=0, 25.16=0,
         9.17=0, 10.17=0, 11.17=100, 12.17=0, 13.17=0, 14.17=0, 15.17=0, 16.17=0, 17.17=0, 18.17=0, 19.17=0, 20.17=0, 21.17=0,
22.17=0, 23.17=0, 24.17=100, 25.17=0,
         10.18=0, 11.18=0, 12.18=0, 13.18=0, 14.18=0, 15.18=0, 16.18=0, 17.18=0, 18.18=100, 19.18=0, 20.18=0, 21.18=0, 22.18=0,
23.18=0, 24.18=0, 25.18=0, 26.18=0,
         10.19=0, 11.19=0, 12.19=0, 13.19=0, 14.19=0, 15.19=0, 16.19=0, 17.19=0, 18.19=0, 19.19=0, 20.19=0, 21.19=0, 22.19=0,
23.19=0, 24.19=0, 25.19=0, 26.19=0,
         11.20=0, 12.20=0, 13.20=0, 14.20=0, 15.20=0, 16.20=0, 17.20=0, 18.20=0, 19.20=0, 20.20=100, 21.20=0, 22.20=0, 23.20=0,
24.20=0, 25.20=0, 26.20=0, 27.20=0,
         11.21=0, 12.21=0, 13.21=0, 14.21=100, 15.21=100, 16.21=0, 17.21=0, 18.21=0, 19.21=0, 20.21=0, 21.21=0, 22.21=0, 23.21=0,
24.21=0, 25.21=100, 26.21=0, 27.21=0,
         12.22=0, 13.22=0, 14.22=0, 15.22=0, 16.22=0, 17.22=0, 18.22=0, 19.22=100, 20.22=0, 21.22=0, 22.22=0, 23.22=0, 24.22=0,
25.22=0, 26.22=0, 27.22=0, 28.22=0,
         12.23=0, 13.23=0, 14.23=0, 15.23=0, 16.23=0, 17.23=0, 18.23=0, 19.23=0, 20.23=0, 21.23=0, 22.23=0, 23.23=0, 24.23=0,
25.23=0, 26.23=0, 27.23=0, 28.23=100,
         13.24=0, 14.24=0, 15.24=0, 16.24=0, 17.24=100, 18.24=0, 19.24=0, 20.24=0, 21.24=0, 22.24=0, 23.24=0, 24.24=100, 25.24=0,
26.24=0, 27.24=0, 28.24=0, 29.24=0,
```

```
        13.25=0, 14.25=0, 15.25=0, 16.25=0, 17.25=0, 18.25=0, 19.25=0, 20.25=0, 21.25=0, 22.25=0, 23.25=0, 24.25=0, 25.25=0,
26.25=0, 27.25=0, 28.25=0, 29.25=0/;
*================================== VARIABLES =================================
BINARY VARIABLE
        X(i,j) node equals 1 if a node is placed at grid i-j and 0 otherwise
        Y(i,j) zone equals 1 if no comms link to a node within range;

VARIABLE
        Z total uncovered importance of zones;

*================================== EQUATIONS =================================
EQUATIONS
        OBJ total uncovered importance
        WITHIN_RANGE(i,j)
        NO GO
        MAX_NODES;

*We seek to minimize the "total importance" of uncovered zones in obj function.
OBJ..
        Z =E= Sum( (i,j), Y(i,j)*importance(i,j) );

*For each zone, there must be at least a node at the zone, or at least one node
*at the 6 adjacent zones, or else Y(i,j) is set to 1, meaning that that
*a particular zone X(i,j) is not covered.
WITHIN_RANGE(i,j)..
          X(i-1,j-1) + X(i,j-1)
        + X(i-1,j) + X(i,j) + X(i+1,j)
        + X(i,j+1) + X(i+1,j+1)
        + Y(i,j) =G=1;

*These are zones that no node is allowed to be deployed
NO GO..
          X("8","1") + X("16","1")
        + X("4","2") + X("7","2")
        + X("12","3")
        + X("14","4") + X("17","4")
        + X("5","6")
        + X("11","7") + X("12","7")
        + X("10","8") + X("19","8")
        + X("17","11")
        + X("9","12")
        + X("15","14")
        + X("20","15")
        + X("16","17")
        + X("16","18") + X("17","18")
        + X("23","19") + X("24","19")
        + X("23","24")
        + X("15","25") =E=0;

*Set the maximum number of nodes available for deployment
MAX_NODES..
        Sum( (i,j), X(i,j) ) =L=30;

*================================== MODEL =====================================
MODEL comms_gs1 /ALL/;
OPTION LP=OSL;
OPTION MIP=XA;
OPTION ITERLIM = 500000;
OPTION RESLIM = 100000;
SOLVE comms_gs1 USING MIP MINIMIZING Z;

*=============================== DISPLAY RESULTS ==============================
DISPLAY
        Z.l
        X.l
        Y.l;

*=================================== END ======================================
```

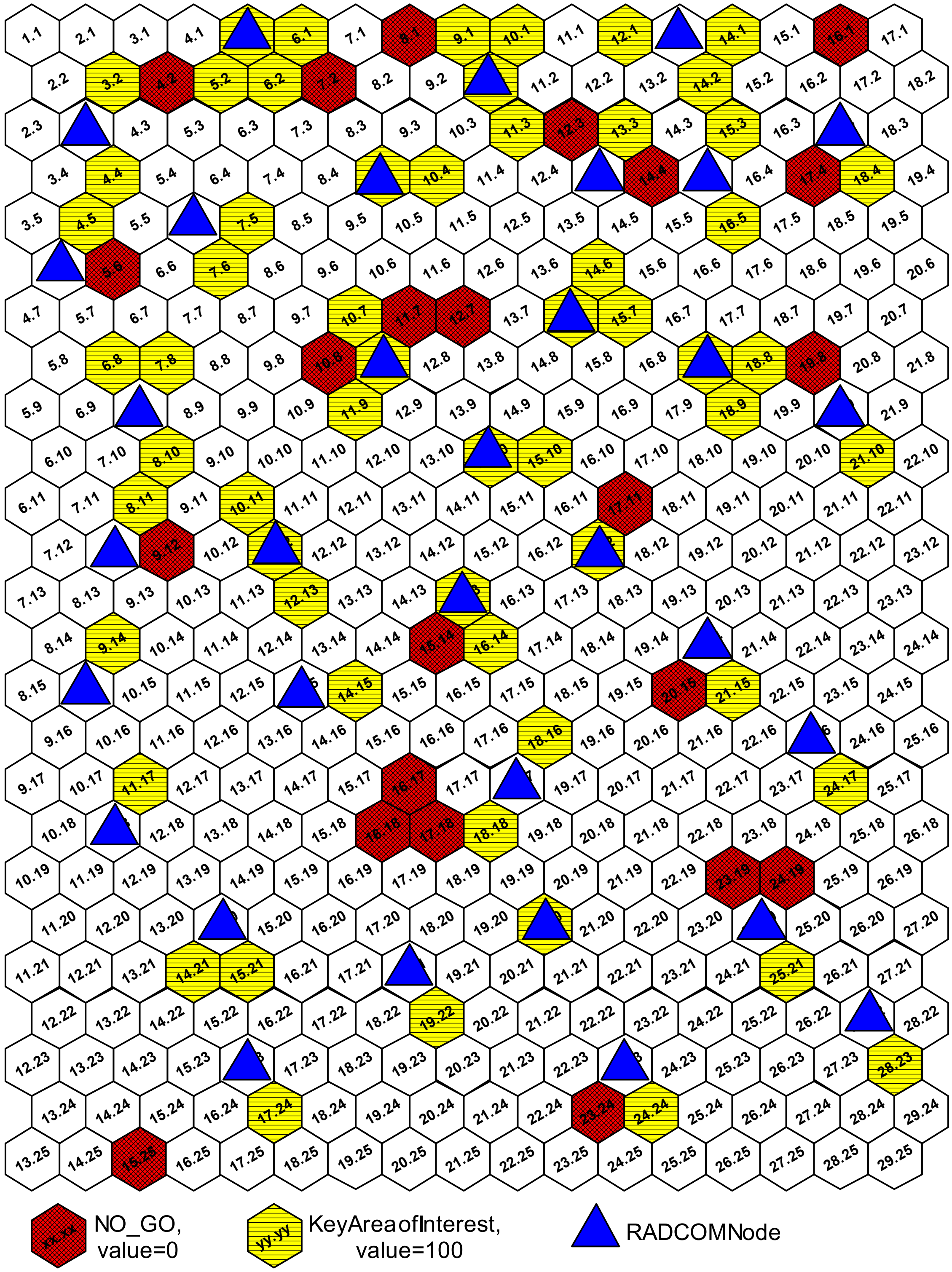

Figure 26. Node placement for Scenario One.

Gams Code for Twice the Number of NO GO Zones

```
*============================ Generic scenario TWO ==============================
*Filename: comms_gs2
*Using importance with just either 0 or 100 and generic grid of 17x25.
*Comms range assumed to be 10km or 5.4NM
*Investigate effects of doubling number of NO GO zones.

*===================================== SETS =====================================
*(1,1) starts from the top left corner
SETS     I column grid number /1*29/
         J row grid number /1*25/;

*================================== PARAMETERS =================================
*Importance of zones, or can look at it as penalty if not covered
*Input values from 0(least important) to 100(most important),
*intermediate values are allowed, but not used in this version.
PARAMETERS       importance(i,j) /
         1.1=0, 2.1=0, 3.1=0, 4.1=0, 5.1=100, 6.1=100, 7.1=0, 8.1=0, 9.1=100, 10.1=100, 11.1=0, 12.1=100, 13.1=0, 14.1=100, 15.1=0,
16.1=0, 17.1=0,
         2.2=0, 3.2=100, 4.2=0, 5.2=100, 6.2=100, 7.2=0, 8.2=0, 9.2=0, 10.2=100, 11.2=0, 12.2=0, 13.2=0, 14.2=100, 15.2=0, 16.2=0,
17.2=0, 18.2=0,
         2.3=0, 3.3=0, 4.3=0, 5.3=0, 6.3=0, 7.3=0, 8.3=0, 9.3=0, 10.3=0, 11.3=100, 12.3=0, 13.3=100, 14.3=0, 15.3=100, 16.3=0,
17.3=0, 18.3=0,
         3.4=0, 4.4=100, 5.4=0, 6.4=0, 7.4=0, 8.4=0, 9.4=100, 10.4=100, 11.4=0, 12.4=0, 13.4=0, 14.4=0, 15.4=0, 16.4=0, 17.4=0,
18.4=100, 19.4=0,
         3.5=0, 4.5=100, 5.5=0, 6.5=0, 7.5=100, 8.5=0, 9.5=0, 10.5=0, 11.5=0, 12.5=0, 13.5=0, 14.5=0, 15.5=0, 16.5=100, 17.5=0,
18.5=0, 19.5=0,
         4.6=0, 5.6=0, 6.6=0, 7.6=100, 8.6=0, 9.6=0, 10.6=0, 11.6=0, 12.6=0, 13.6=0, 14.6=100, 15.6=0, 16.6=0, 17.6=0, 18.6=0,
19.6=0, 20.6=0,
         4.7=0, 5.7=0, 6.7=0, 7.7=0, 8.7=0, 9.7=0, 10.7=100, 11.7=0, 12.7=0, 13.7=0, 14.7=100, 15.7=100, 16.7=0, 17.7=0, 18.7=0,
19.7=0, 20.7=0,
         5.8=0, 6.8=100, 7.8=100, 8.8=0, 9.8=0, 10.8=0, 11.8=100, 12.8=0, 13.8=0, 14.8=0, 15.8=0, 16.8=0, 17.8=100, 18.8=100,
19.8=0, 20.8=0, 21.8=0,
         5.9=0, 6.9=0, 7.9=0, 8.9=0, 9.9=0, 10.9=0, 11.9=100, 12.9=0, 13.9=0, 14.9=0, 15.9=0, 16.9=0, 17.9=0, 18.9=100, 19.9=0,
20.9=0, 21.9=0,
         6.10=0, 7.10=0, 8.10=100, 9.10=0, 10.10=0, 11.10=0, 12.10=0, 13.10=0, 14.10=100, 15.10=100, 16.10=0, 17.10=0, 18.10=0,
19.10=0, 20.10=0, 21.10=100, 22.10=0,
         6.11=0, 7.11=0, 8.11=100, 9.11=0, 10.11=100, 11.11=0, 12.11=0, 13.11=0, 14.11=0, 15.11=0, 16.11=0, 17.11=0, 18.11=0,
19.11=0, 20.11=0, 21.11=0, 22.11=0,
         7.12=0, 8.12=0, 9.12=0, 10.12=0, 11.12=100, 12.12=0, 13.12=0, 14.12=0, 15.12=0, 16.12=0, 17.12=100, 18.12=0, 19.12=0,
20.12=0, 21.12=0, 22.12=0, 23.12=0,
         7.13=0, 8.13=0, 9.13=0, 10.13=0, 11.13=0, 12.13=100, 13.13=0, 14.13=0, 15.13=100, 16.13=0, 17.13=0, 18.13=0, 19.13=0,
20.13=0, 21.13=0, 22.13=0, 23.13=0,
         8.14=0, 9.14=100, 10.14=0, 11.14=0, 12.14=0, 13.14=0, 14.14=0, 15.14=0, 16.14=100, 17.14=0, 18.14=0, 19.14=0, 20.14=0,
21.14=0, 22.14=0, 23.14=0, 24.14=0,
         8.15=0, 9.15=0, 10.15=0, 11.15=0, 12.15=0, 13.15=0, 14.15=100, 15.15=0, 16.15=0, 17.15=0, 18.15=0, 19.15=0, 20.15=0,
21.15=100, 22.15=0, 23.15=0, 24.15=0,
         9.16=0, 10.16=0, 11.16=0, 12.16=0, 13.16=0, 14.16=0, 15.16=0, 16.16=0, 17.16=0, 18.16=100, 19.16=0, 20.16=0, 21.16=0,
22.16=0, 23.16=0, 24.16=0, 25.16=0,
         9.17=0, 10.17=0, 11.17=100, 12.17=0, 13.17=0, 14.17=0, 15.17=0, 16.17=0, 17.17=0, 18.17=0, 19.17=0, 20.17=0, 21.17=0,
22.17=0, 23.17=0, 24.17=100, 25.17=0,
         10.18=0, 11.18=0, 12.18=0, 13.18=0, 14.18=0, 15.18=0, 16.18=0, 17.18=0, 18.18=100, 19.18=0, 20.18=0, 21.18=0, 22.18=0,
23.18=0, 24.18=0, 25.18=0, 26.18=0,
         10.19=0, 11.19=0, 12.19=0, 13.19=0, 14.19=0, 15.19=0, 16.19=0, 17.19=0, 18.19=0, 19.19=0, 20.19=0, 21.19=0, 22.19=0,
23.19=0, 24.19=0, 25.19=0, 26.19=0,
         11.20=0, 12.20=0, 13.20=0, 14.20=0, 15.20=0, 16.20=0, 17.20=0, 18.20=0, 19.20=0, 20.20=100, 21.20=0, 22.20=0, 23.20=0,
24.20=0, 25.20=0, 26.20=0, 27.20=0,
         11.21=0, 12.21=0, 13.21=0, 14.21=100, 15.21=100, 16.21=0, 17.21=0, 18.21=0, 19.21=0, 20.21=0, 21.21=0, 22.21=0, 23.21=0,
24.21=0, 25.21=100, 26.21=0, 27.21=0,
         12.22=0, 13.22=0, 14.22=0, 15.22=0, 16.22=0, 17.22=0, 18.22=0, 19.22=100, 20.22=0, 21.22=0, 22.22=0, 23.22=0, 24.22=0,
25.22=0, 26.22=0, 27.22=0, 28.22=0,
         12.23=0, 13.23=0, 14.23=0, 15.23=0, 16.23=0, 17.23=0, 18.23=0, 19.23=0, 20.23=0, 21.23=0, 22.23=0, 23.23=0, 24.23=0,
25.23=0, 26.23=0, 27.23=0, 28.23=100,
         13.24=0, 14.24=0, 15.24=0, 16.24=0, 17.24=100, 18.24=0, 19.24=0, 20.24=0, 21.24=0, 22.24=0, 23.24=0, 24.24=100, 25.24=0,
26.24=0, 27.24=0, 28.24=0, 29.24=0,
         13.25=0, 14.25=0, 15.25=0, 16.25=0, 17.25=0, 18.25=0, 19.25=0, 20.25=0, 21.25=0, 22.25=0, 23.25=0, 24.25=0, 25.25=0,
26.25=0, 27.25=0, 28.25=0, 29.25=0/;
```

```
*================================== VARIABLES =================================
BINARY VARIABLE
        X(i,j) node equals 1 if a node is placed at grid i-j and 0 otherwise
        Y(i,j) zone equals 1 if no comms link to a node within range;

VARIABLE
        Z total uncovered importance of zones;

*================================== EQUATIONS =================================
EQUATIONS
        OBJ total uncovered importance
        WITHIN_RANGE(i,j)
        NO GO
        MAX_NODES;

*We seek to minimize the "total importance" of uncovered zones in obj function.
OBJ..
        Z =E= Sum( (i,j), Y(i,j)*importance(i,j) );

*For each zone, there must be at least a node at the zone, or at least one node
*at the 6 adjacent zones, or else Y(i,j) is set to 1, meaning that that
*a particular zone X(i,j) is not covered.
WITHIN_RANGE(i,j)..
          X(i-1,j-1) + X(i,j-1)
        + X(i-1,j) + X(i,j) + X(i+1,j)
        + X(i,j+1) + X(i+1,j+1)
        + Y(i,j) =G=1;

*These are zones that no node is allowed to be deployed
NO GO..
          X("8","1") + X("15","1") + X("16","1")
        + X("4","2") + X("7","2") + X("9","2") + X("12","2")
        + X("4","3") + X("8","3") + X("12","3")+ X("14","3")
        + X("14","4") + X("17","4")+ X("16","4")
        + X("5","6") + X("6","6") + X("10","6") + X("11","6")
        + X("9","7") + X("11","7") + X("12","7")
        + X("10","8") + X("19","8")
        + X("19","9")
        + X("16","11") + X("17","11")
        + X("8","12") + X("9","12")
        + X("15","14")
        + X("16","15") + X("20","15")
        + X("21","16")
        + X("16","17") + X("17","17")
        + X("16","18") + X("17","18") + X("23","18") + X("24","18")
        + X("17","19") + X("18","19") + X("23","19") + X("24","19")
        + X("23","23")
        + X("23","24")
        + X("15","24") + X("15","25") =E=0;

*Set the maximum number of nodes available for deployment
MAX_NODES..
        Sum( (i,j), X(i,j) ) =L=30;

*================================== MODEL ====================================
MODEL comms_gs2 /ALL/;
OPTION LP=OSL;
OPTION MIP=XA;
OPTION ITERLIM = 500000;
OPTION RESLIM = 100000;
SOLVE comms_gs2 USING MIP MINIMIZING Z;

*=============================== DISPLAY RESULTS ==============================
DISPLAY
        Z.l
        X.l
        Y.l;

*==================================== END =====================================
```

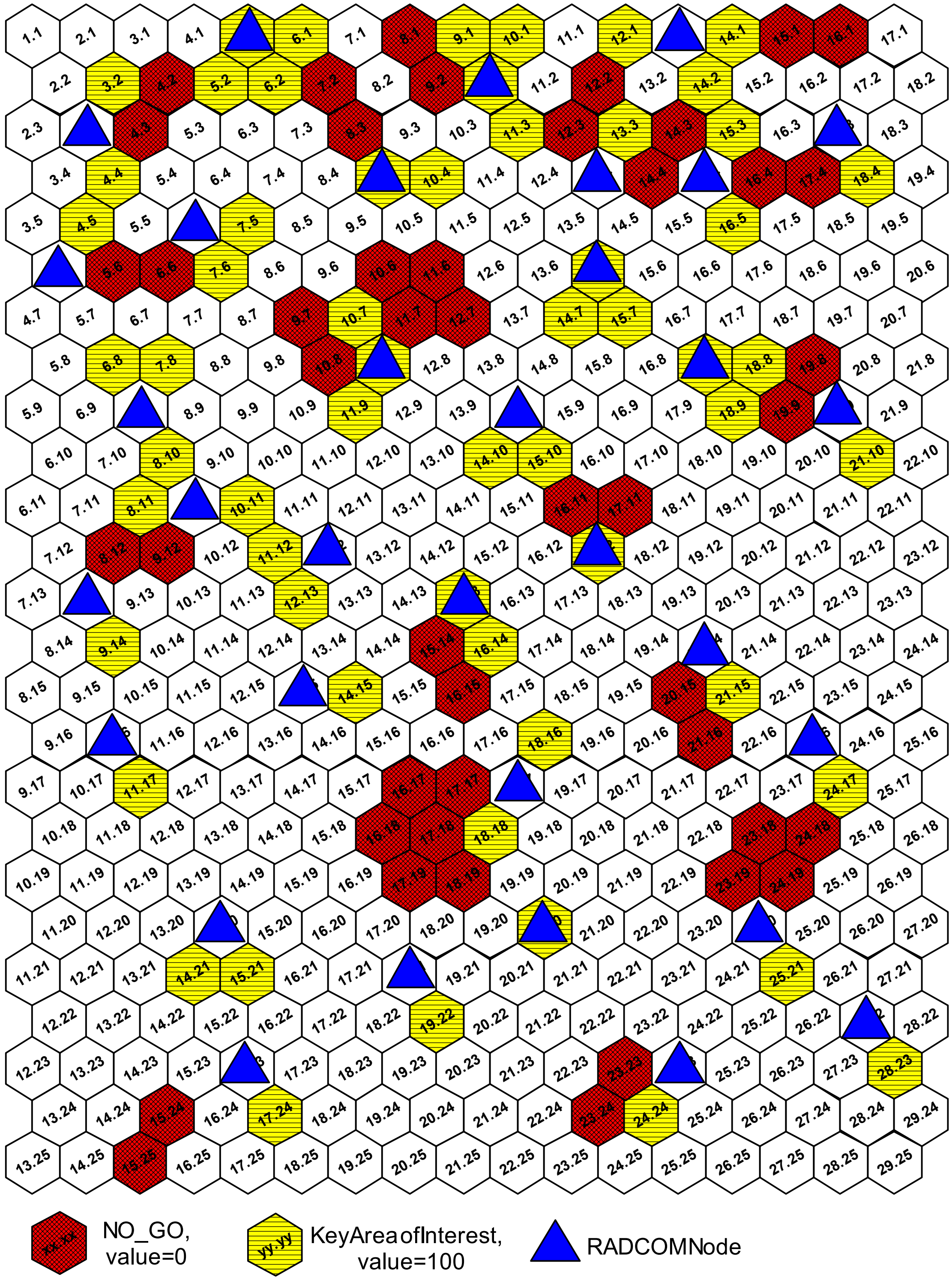

Figure 27. Node placement for Scenario Two.

GAMS Code for Scenario Two

```
*=========================== Generic scenario THREE ============================
*Filename: comms_gs3
*Using importance with just either 0 or 100 and generic grid of 17x25.
*Comms range assumed to be 10km or 5.4NM
*Investigate effects of imposing no go restriction on all key areas of interest.

*==================================== SETS ====================================
*(1,1) starts from the top left corner
SETS    I column grid number /1*29/
        J row grid number /1*25/;

*================================== PARAMETERS ================================
*Importance of zones, or can look at it as penalty if not covered
*Input values from 0(least important) to 100(most important),
*intermediate values are allowed, but not used in this version.
PARAMETERS      importance(i,j) /
        1.1=0, 2.1=0, 3.1=0, 4.1=0, 5.1=100, 6.1=100, 7.1=0, 8.1=0, 9.1=100, 10.1=100, 11.1=0, 12.1=100, 13.1=0, 14.1=100, 15.1=0,
16.1=0, 17.1=0,
        2.2=0, 3.2=100, 4.2=0, 5.2=100, 6.2=100, 7.2=0, 8.2=0, 9.2=0, 10.2=100, 11.2=0, 12.2=0, 13.2=0, 14.2=100, 15.2=0, 16.2=0,
17.2=0, 18.2=0,
        2.3=0, 3.3=0, 4.3=0, 5.3=0, 6.3=0, 7.3=0, 8.3=0, 9.3=0, 10.3=0, 11.3=100, 12.3=0, 13.3=100, 14.3=0, 15.3=100, 16.3=0,
17.3=0, 18.3=0,
        3.4=0, 4.4=100, 5.4=0, 6.4=0, 7.4=0, 8.4=0, 9.4=100, 10.4=100, 11.4=0, 12.4=0, 13.4=0, 14.4=0, 15.4=0, 16.4=0, 17.4=0,
18.4=100, 19.4=0,
        3.5=0, 4.5=100, 5.5=0, 6.5=0, 7.5=100, 8.5=0, 9.5=0, 10.5=0, 11.5=0, 12.5=0, 13.5=0, 14.5=0, 15.5=0, 16.5=100, 17.5=0,
18.5=0, 19.5=0,
        4.6=0, 5.6=0, 6.6=0, 7.6=100, 8.6=0, 9.6=0, 10.6=0, 11.6=0, 12.6=0, 13.6=0, 14.6=100, 15.6=0, 16.6=0, 17.6=0, 18.6=0,
19.6=0, 20.6=0,
        4.7=0, 5.7=0, 6.7=0, 7.7=0, 8.7=0, 9.7=0, 10.7=100, 11.7=0, 12.7=0, 13.7=0, 14.7=100, 15.7=100, 16.7=0, 17.7=0, 18.7=0,
19.7=0, 20.7=0,
        5.8=0, 6.8=100, 7.8=100, 8.8=0, 9.8=0, 10.8=0, 11.8=100, 12.8=0, 13.8=0, 14.8=0, 15.8=0, 16.8=0, 17.8=100, 18.8=100,
19.8=0, 20.8=0, 21.8=0,
        5.9=0, 6.9=0, 7.9=0, 8.9=0, 9.9=0, 10.9=0, 11.9=100, 12.9=0, 13.9=0, 14.9=0, 15.9=0, 16.9=0, 17.9=0, 18.9=100, 19.9=0,
20.9=0, 21.9=0,
        6.10=0, 7.10=0, 8.10=100, 9.10=0, 10.10=0, 11.10=0, 12.10=0, 13.10=0, 14.10=100, 15.10=100, 16.10=0, 17.10=0, 18.10=0,
19.10=0, 20.10=0, 21.10=100, 22.10=0,
        6.11=0, 7.11=0, 8.11=100, 9.11=0, 10.11=100, 11.11=0, 12.11=0, 13.11=0, 14.11=0, 15.11=0, 16.11=0, 17.11=0, 18.11=0,
19.11=0, 20.11=0, 21.11=0, 22.11=0,
        7.12=0, 8.12=0, 9.12=0, 10.12=0, 11.12=100, 12.12=0, 13.12=0, 14.12=0, 15.12=0, 16.12=0, 17.12=100, 18.12=0, 19.12=0,
20.12=0, 21.12=0, 22.12=0, 23.12=0,
        7.13=0, 8.13=0, 9.13=0, 10.13=0, 11.13=0, 12.13=100, 13.13=0, 14.13=0, 15.13=100, 16.13=0, 17.13=0, 18.13=0, 19.13=0,
20.13=0, 21.13=0, 22.13=0, 23.13=0,
        8.14=0, 9.14=100, 10.14=0, 11.14=0, 12.14=0, 13.14=0, 14.14=0, 15.14=0, 16.14=100, 17.14=0, 18.14=0, 19.14=0, 20.14=0,
21.14=0, 22.14=0, 23.14=0, 24.14=0,
        8.15=0, 9.15=0, 10.15=0, 11.15=0, 12.15=0, 13.15=0, 14.15=100, 15.15=0, 16.15=0, 17.15=0, 18.15=0, 19.15=0, 20.15=0,
21.15=100, 22.15=0, 23.15=0, 24.15=0,
        9.16=0, 10.16=0, 11.16=0, 12.16=0, 13.16=0, 14.16=0, 15.16=0, 16.16=0, 17.16=0, 18.16=100, 19.16=0, 20.16=0, 21.16=0,
22.16=0, 23.16=0, 24.16=0, 25.16=0,
        9.17=0, 10.17=0, 11.17=100, 12.17=0, 13.17=0, 14.17=0, 15.17=0, 16.17=0, 17.17=0, 18.17=0, 19.17=0, 20.17=0, 21.17=0,
22.17=0, 23.17=0, 24.17=100, 25.17=0,
        10.18=0, 11.18=0, 12.18=0, 13.18=0, 14.18=0, 15.18=0, 16.18=0, 17.18=0, 18.18=100, 19.18=0, 20.18=0, 21.18=0, 22.18=0,
23.18=0, 24.18=0, 25.18=0, 26.18=0,
        10.19=0, 11.19=0, 12.19=0, 13.19=0, 14.19=0, 15.19=0, 16.19=0, 17.19=0, 18.19=0, 19.19=0, 20.19=0, 21.19=0, 22.19=0,
23.19=0, 24.19=0, 25.19=0, 26.19=0,
        11.20=0, 12.20=0, 13.20=0, 14.20=0, 15.20=0, 16.20=0, 17.20=0, 18.20=0, 19.20=0, 20.20=100, 21.20=0, 22.20=0, 23.20=0,
24.20=0, 25.20=0, 26.20=0, 27.20=0,
        11.21=0, 12.21=0, 13.21=0, 14.21=100, 15.21=100, 16.21=0, 17.21=0, 18.21=0, 19.21=0, 20.21=0, 21.21=0, 22.21=0, 23.21=0,
24.21=0, 25.21=100, 26.21=0, 27.21=0,
        12.22=0, 13.22=0, 14.22=0, 15.22=0, 16.22=0, 17.22=0, 18.22=0, 19.22=100, 20.22=0, 21.22=0, 22.22=0, 23.22=0, 24.22=0,
25.22=0, 26.22=0, 27.22=0, 28.22=0,
        12.23=0, 13.23=0, 14.23=0, 15.23=0, 16.23=0, 17.23=0, 18.23=0, 19.23=0, 20.23=0, 21.23=0, 22.23=0, 23.23=0, 24.23=0,
25.23=0, 26.23=0, 27.23=0, 28.23=100,
        13.24=0, 14.24=0, 15.24=0, 16.24=0, 17.24=100, 18.24=0, 19.24=0, 20.24=0, 21.24=0, 22.24=0, 23.24=0, 24.24=100, 25.24=0,
26.24=0, 27.24=0, 28.24=0, 29.24=0,
        13.25=0, 14.25=0, 15.25=0, 16.25=0, 17.25=0, 18.25=0, 19.25=0, 20.25=0, 21.25=0, 22.25=0, 23.25=0, 24.25=0, 25.25=0,
26.25=0, 27.25=0, 28.25=0, 29.25=0/;
```

```
*================================== VARIABLES =================================
BINARY VARIABLE
        X(i,j) node equals 1 if a node is placed at grid i-j and 0 otherwise
        Y(i,j) zone equals 1 if no comms link to a node within range;

VARIABLE
        Z total uncovered importance of zones;

*================================== EQUATIONS =================================
EQUATIONS
        OBJ total uncovered importance
        WITHIN_RANGE(i,j)
        NO GO
        MAX_NODES;

*We seek to minimize the "total importance" of uncovered zones in obj function.
OBJ..
        Z =E= Sum( (i,j), Y(i,j)*importance(i,j) );

*For each zone, there must be at least a node at the zone, or at least one node
*at the 6 adjacent zones, or else Y(i,j) is set to 1, meaning that that
*a particular zone X(i,j) is not covered.
WITHIN_RANGE(i,j)..
          X(i-1,j-1) + X(i,j-1)
        + X(i-1,j) + X(i,j) + X(i+1,j)
        + X(i,j+1) + X(i+1,j+1)
        + Y(i,j) =G=1;

*These are zones that no node is allowed to be deployed
NO GO..
          X("5","1") + X("6","1") + X("8","1") + X("9","1") + X("10","1") + X("12","1") + X("14","1") + X("16","1")
        + X("3","2") + X("4","2") + X("5","2") + X("6","2") + X("7","2") + X("10","2") + X("4","2") + X("14","2")
        + X("11","3")+ X("12","3") + X("13","3") + X("15","3")
        + X("4","4")+ X("9","4") + X("10","4") + X("14","4") + X("17","4") + X("18","4")
        + X("4","5") + X("7","5") + X("16","5")
        + X("5","6") + X("7","6") + X("14","6")
        + X("10","7")+ X("11","7") + X("12","7") + X("14","7") + X("15","7")
        + X("6","8")+ X("7","8") + X("10","8") + X("11","8") + X("17","8") + X("18","8") + X("19","8")
        + X("11","9") + X("18","9")
        + X("8","10") + X("14","10") + X("15","10") + X("21","10")
        + X("8","11") + X("10","11")+ X("17","11")
        + X("9","12") + X("11","12") + X("17","12")
        + X("12","13") + X("15","13")
        + X("9","14") + X("15","14") + X("16","14")
        + X("14","15") + X("20","15") + X("21","15")
        + X("18","16")
        + X("11","17") + X("16","17") + X("24","17")
        + X("16","18") + X("17","18") + X("18","18")
        + X("23","19") + X("24","19")
        + X("20","20")
        + X("14","21") + X("15","21") + X("25","21")
        + X("19","22")
        + X("28","23")
        + X("17","24") + X("23","24") + X("24","24")
        + X("15","25") =E=0;

*Set the maximum number of nodes available for deployment
MAX_NODES..
        Sum( (i,j), X(i,j) ) =L=30;

*================================== MODEL =====================================
MODEL comms_gs3 /ALL/;
OPTION LP=OSL;
OPTION MIP=XA;
OPTION ITERLIM = 500000;
OPTION RESLIM = 100000;
SOLVE comms_gs3 USING MIP MINIMIZING Z;
```

```
*================================ DISPLAY RESULTS ==============================
DISPLAY
        Z.l
        X.l
        Y.l;

*==================================== END ======================================
```

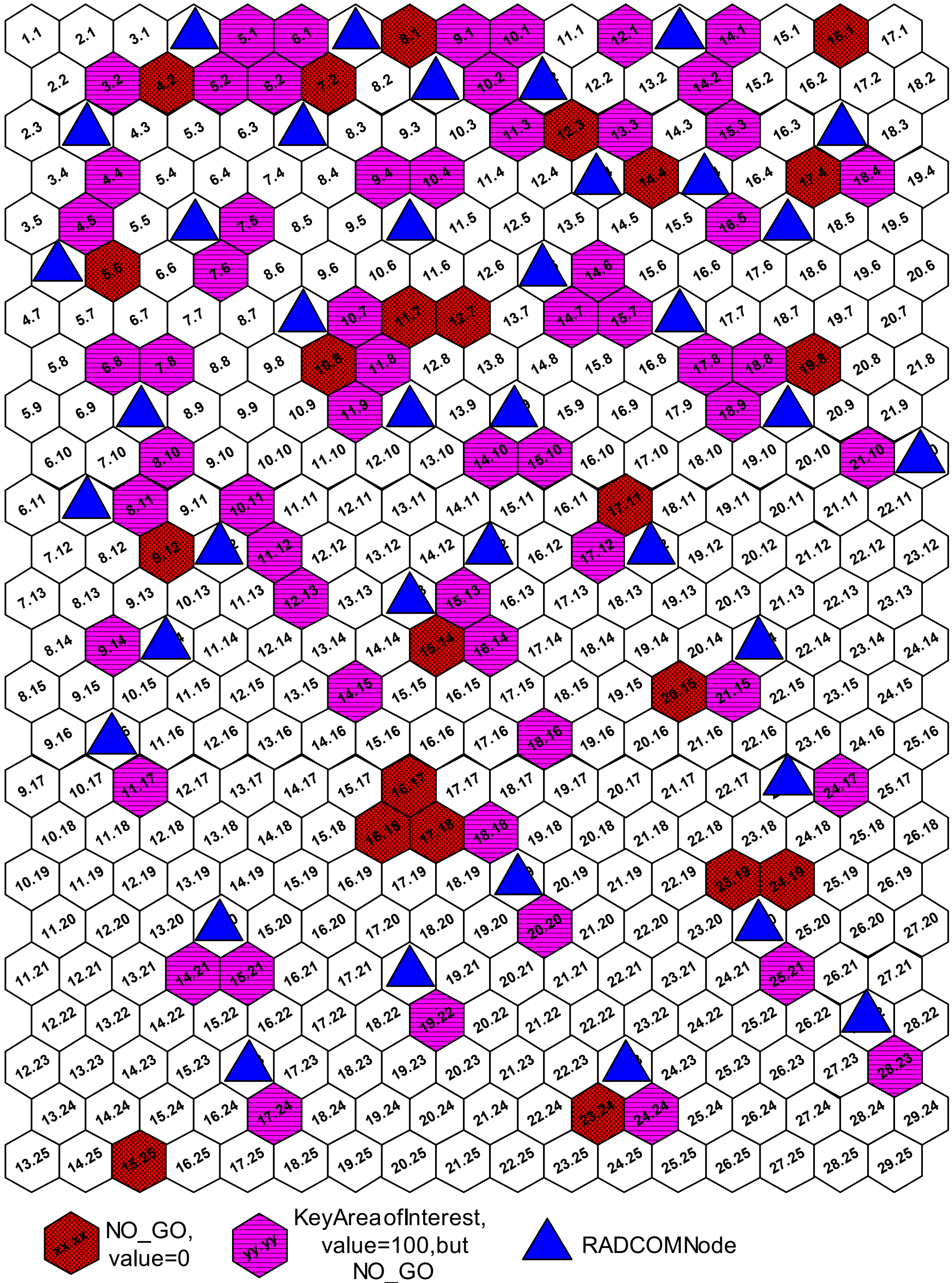

Figure 28. Node placement for Scenario Three.

GAMS Code for Computing Primary and Secondary Areas of Interest

```
*=========================== Generic scenario FOUR =============================
*Filename: comms_gs4
*Using importance with just either 0, 70 or 100 and generic grid of 17x25.
*Comms range assumed to be 10km or 5.4NM
*Investigate effects of adding secondary areas of interest.

*=================================== SETS ===================================
*(1,1) starts from the top left corner
SETS     I column grid number /1*29/
        J row grid number /1*25/;

*================================= PARAMETERS ================================
*Importance of zones, or can look at it as penalty if not covered
*Input values from 0(least important) to 100(most important),
*intermediate values are allowed.
PARAMETERS       importance(i,j) /
        1.1=0, 2.1=70, 3.1=70, 4.1=70, 5.1=100, 6.1=100, 7.1=70, 8.1=70, 9.1=100, 10.1=100, 11.1=70, 12.1=100, 13.1=70, 14.1=100,
15.1=70, 16.1=0, 17.1=0,
        2.2=70, 3.2=100, 4.2=70, 5.2=100, 6.2=100, 7.2=70, 8.2=0, 9.2=70, 10.2=100, 11.2=70, 12.2=70, 13.2=70, 14.2=100, 15.2=70,
16.2=0, 17.2=0, 18.2=0,
        2.3=0, 3.3=70, 4.3=70, 5.3=70, 6.3=70, 7.3=70, 8.3=70, 9.3=70, 10.3=70, 11.3=100, 12.3=0, 13.3=100, 14.3=70, 15.3=100,
16.3=70, 17.3=70, 18.3=70,
        3.4=70, 4.4=100, 5.4=70, 6.4=70, 7.4=70, 8.4=70, 9.4=100, 10.4=100, 11.4=70, 12.4=0, 13.4=0, 14.4=0, 15.4=70, 16.4=70,
17.4=0, 18.4=100, 19.4=70,
        3.5=70, 4.5=100, 5.5=70, 6.5=70, 7.5=100, 8.5=70, 9.5=70, 10.5=70, 11.5=70, 12.5=0, 13.5=70, 14.5=70, 15.5=70, 16.5=100,
17.5=70, 18.5=70, 19.5=70,
        4.6=70, 5.6=0, 6.6=70, 7.6=100, 8.6=70, 9.6=70, 10.6=70, 11.6=0, 12.6=0, 13.6=70, 14.6=100, 15.6=70, 16.6=70, 17.6=70,
18.6=0, 19.6=0, 20.6=0,
        4.7=0, 5.7=70, 6.7=70, 7.7=70, 8.7=70, 9.7=70, 10.7=100, 11.7=0, 12.7=0, 13.7=70, 14.7=100, 15.7=100, 16.7=70, 17.7=70,
18.7=70, 19.7=0, 20.7=0,
        5.8=70, 6.8=100, 7.8=100, 8.8=70, 9.8=0, 10.8=0, 11.8=100, 12.8=70, 13.8=0, 14.8=70, 15.8=70, 16.8=70, 17.8=100, 18.8=100,
19.8=70, 20.8=0, 21.8=0,
        5.9=0, 6.9=70, 7.9=70, 8.9=70, 9.9=0, 10.9=70, 11.9=100, 12.9=70, 13.9=70, 14.9=70, 15.9=70, 16.9=0, 17.9=70, 18.9=100,
19.9=70, 20.9=70, 21.9=70,
        6.10=0, 7.10=70, 8.10=100, 9.10=70, 10.10=70, 11.10=70, 12.10=70, 13.10=70, 14.10=100, 15.10=100, 16.10=70, 17.10=0,
18.10=70, 19.10=70, 20.10=70, 21.10=100, 22.10=70,
        6.11=0, 7.11=70, 8.11=100, 9.11=70, 10.11=100, 11.11=70, 12.11=0, 13.11=0, 14.11=70, 15.11=70, 16.11=70, 17.11=0,
18.11=0, 19.11=0, 20.11=0, 21.11=70, 22.11=70,
        7.12=0, 8.12=70, 9.12=0, 10.12=70, 11.12=100, 12.12=70, 13.12=0, 14.12=70, 15.12=70, 16.12=70, 17.12=100, 18.12=70,
19.12=0, 20.12=0, 21.12=0, 22.12=0, 23.12=0,
        7.13=0, 8.13=70, 9.13=70, 10.13=0, 11.13=70, 12.13=100, 13.13=70, 14.13=70, 15.13=100, 16.13=70, 17.13=70, 18.13=70,
19.13=0, 20.13=0, 21.13=0, 22.13=0, 23.13=0,
        8.14=70, 9.14=100, 10.14=70, 11.14=0, 12.14=70, 13.14=70, 14.14=70, 15.14=0, 16.14=100, 17.14=70, 18.14=0, 19.14=0,
20.14=70, 21.14=70, 22.14=0, 23.14=0, 24.14=0,
        8.15=0, 9.15=70, 10.15=70, 11.15=0, 12.15=0, 13.15=70, 14.15=100, 15.15=70, 16.15=70, 17.15=70, 18.15=70, 19.15=0,
20.15=70, 21.15=100, 22.15=70, 23.15=0, 24.15=0,
        9.16=0, 10.16=70, 11.16=70, 12.16=0, 13.16=0, 14.16=70, 15.16=70, 16.16=0, 17.16=70, 18.16=100, 19.16=70, 20.16=0,
21.16=70, 22.16=70, 23.16=70, 24.16=70, 25.16=0,
        9.17=0, 10.17=70, 11.17=100, 12.17=70, 13.17=0, 14.17=0, 15.17=0, 16.17=70, 17.17=70, 18.17=70, 19.17=70, 20.17=0,
21.17=0, 22.17=0, 23.17=70, 24.17=100, 25.17=70,
        10.18=0, 11.18=70, 12.18=70, 13.18=0, 14.18=0, 15.18=0, 16.18=0, 17.18=0, 18.18=100, 19.18=70, 20.18=0, 21.18=0,
22.18=0, 23.18=0, 24.18=70, 25.18=70, 26.18=0,
        10.19=0, 11.19=0, 12.19=0, 13.19=0, 14.19=0, 15.19=0, 16.19=0, 17.19=0, 18.19=70, 19.19=70, 20.19=70, 21.19=0, 22.19=0,
23.19=0, 24.19=0, 25.19=0, 26.19=0,
        11.20=0, 12.20=0, 13.20=70, 14.20=70, 15.20=70, 16.20=0, 17.20=0, 18.20=0, 19.20=70, 20.20=100, 21.20=70, 22.20=0,
23.20=0, 24.20=70, 25.20=70, 26.20=0, 27.20=0,
        11.21=0, 12.21=0, 13.21=70, 14.21=100, 15.21=100, 16.21=70, 17.21=0, 18.21=70, 19.21=70, 20.21=70, 21.21=70, 22.21=0,
23.21=0, 24.21=70, 25.21=100, 26.21=70, 27.21=0,
        12.22=0, 13.22=0, 14.22=70, 15.22=70, 16.22=70, 17.22=0, 18.22=70, 19.22=100, 20.22=70, 21.22=0, 22.22=0, 23.22=0,
24.22=0, 25.22=70, 26.22=70, 27.22=70, 28.22=70,
        12.23=0, 13.23=0, 14.23=0, 15.23=0, 16.23=70, 17.23=70, 18.23=0, 19.23=70, 20.23=70, 21.23=0, 22.23=0, 23.23=70,
24.23=70, 25.23=0, 26.23=0, 27.23=70, 28.23=100,
        13.24=0, 14.24=0, 15.24=0, 16.24=70, 17.24=100, 18.24=70, 19.24=0, 20.24=0, 21.24=0, 22.24=0, 23.24=0, 24.24=100,
25.24=70, 26.24=0, 27.24=0, 28.24=70, 29.24=70,
        13.25=0, 14.25=0, 15.25=0, 16.25=0, 17.25=70, 18.25=70, 19.25=0, 20.25=0, 21.25=0, 22.25=0, 23.25=0, 24.25=70, 25.25=70,
26.25=0, 27.25=0, 28.25=0, 29.25=0/;
```

```
*================================== VARIABLES ================================
BINARY VARIABLE
        X(i,j) node equals 1 if a node is placed at grid i-j and 0 otherwise
        Y(i,j) zone equals 1 if no comms link to a node within range;

VARIABLE
        Z total uncovered importance of zones;

*================================== EQUATIONS ================================
EQUATIONS
        OBJ total uncovered importance
        WITHIN_RANGE(i,j)
        NO GO
        MAX_NODES;

*We seek to minimize the "total importance" of uncovered zones in obj function.
OBJ..
        Z =E= Sum( (i,j), Y(i,j)*importance(i,j) );

*For each zone, there must be at least a node at the zone, or at least one node
*at the 6 adjacent zones, or else Y(i,j) is set to 1, meaning that that
*a particular zone X(i,j) is not covered.
WITHIN_RANGE(i,j)..
          X(i-1,j-1) + X(i,j-1)
        + X(i-1,j) + X(i,j) + X(i+1,j)
        + X(i,j+1) + X(i+1,j+1)
        + Y(i,j) =G=1;

*These are zones that no node is allowed to be deployed
NO GO..
          X("8","1") + X("16","1")
        + X("4","2") + X("7","2")
        + X("12","3")
        + X("14","4") + X("17","4")
        + X("5","6")
        + X("11","7") + X("12","7")
        + X("10","8") + X("19","8")
        + X("17","11")
        + X("9","12")
        + X("15","14")
        + X("20","15")
        + X("16","17")
        + X("16","18") + X("17","18")
        + X("23","19") + X("24","19")
        + X("23","24")
        + X("15","25") =E=0;

*Set the maximum number of nodes available for deployment
MAX_NODES..
        Sum( (i,j), X(i,j) ) =L=30;

*================================== MODEL ====================================
MODEL comms_gs4 /ALL/;
OPTION LP=OSL;
OPTION MIP=XA;
OPTION ITERLIM = 500000;
OPTION RESLIM = 100000;
SOLVE comms_gs4 USING MIP MINIMIZING Z;

*=============================== DISPLAY RESULTS ============================
DISPLAY
        Z.l
        X.l
        Y.l;

*==================================== END ====================================
```

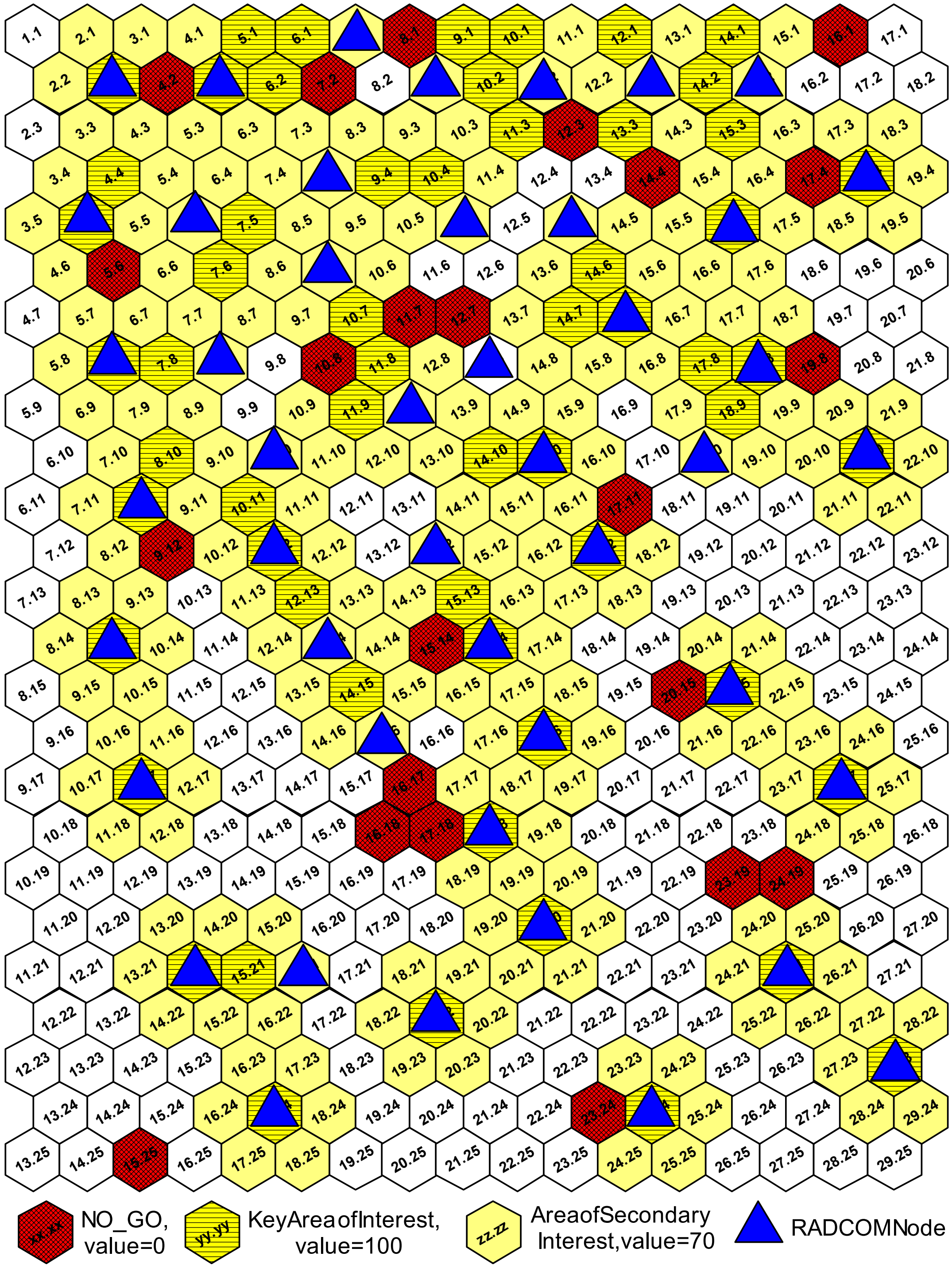

Figure 29. Node placement for Scenario Four.

GAMS Code for Scenario Five

```
*=========================== Generic scenario FIVE ============================
*Filename: comms_gs5
*Using importance with just either 0, 30(area of interest),
*70(area of secondary interest) and 100 and generic grid of 17x25.
*Comms range assumed to be 10km or 5.4NM
*.
*Investigate effects of adding secondary areas of interest, and additional
*constraint where selected few key areas of interest are also no_go.

*==================================== SETS ====================================
*(1,1) starts from the top left corner
SETS     I column grid number /1*29/
         J row grid number /1*25/;

*================================== PARAMETERS ================================
*Importance of zones, or can look at it as penalty if not covered
*Input values from 0(least important) to 100(most important), intermediate
*values are allowed. General rule for assigning area of secondary interest is
*all adjacent zones to a zone of 100 will be given a value of 70 along likely
*route of advance. Other areas are given value of 30.
PARAMETERS       importance(i,j) /
         1.1=0, 2.1=30, 3.1=30, 4.1=30, 5.1=100, 6.1=100, 7.1=30, 8.1=30, 9.1=100, 10.1=100, 11.1=70, 12.1=100, 13.1=70, 14.1=100,
15.1=70, 16.1=0, 17.1=0,
         2.2=30, 3.2=100, 4.2=30, 5.2=100, 6.2=100, 7.2=30, 8.2=0, 9.2=70, 10.2=100, 11.2=70, 12.2=70, 13.2=70, 14.2=100, 15.2=70,
16.2=0, 17.2=0, 18.2=0,
         2.3=0, 3.3=30, 4.3=30, 5.3=30, 6.3=30, 7.3=30, 8.3=70, 9.3=70, 10.3=70, 11.3=100, 12.3=0, 13.3=100, 14.3=70, 15.3=100,
16.3=70, 17.3=30, 18.3=30,
         3.4=30, 4.4=100, 5.4=30, 6.4=30, 7.4=30, 8.4=70, 9.4=100, 10.4=100, 11.4=70, 12.4=0, 13.4=0, 14.4=0, 15.4=70, 16.4=70,
17.4=0, 18.4=100, 19.4=30,
         3.5=30, 4.5=100, 5.5=30, 6.5=30, 7.5=100, 8.5=30, 9.5=70, 10.5=70, 11.5=70, 12.5=0, 13.5=70, 14.5=70, 15.5=30, 16.5=100,
17.5=30, 18.5=30, 19.5=30,
         4.6=30, 5.6=0, 6.6=30, 7.6=100, 8.6=30, 9.6=30, 10.6=30, 11.6=0, 12.6=0, 13.6=70, 14.6=100, 15.6=70, 16.6=30, 17.6=30,
18.6=0, 19.6=0, 20.6=0,
         4.7=0, 5.7=30, 6.7=30, 7.7=30, 8.7=30, 9.7=30, 10.7=100, 11.7=0, 12.7=0, 13.7=70, 14.7=100, 15.7=100, 16.7=70, 17.7=30,
18.7=30, 19.7=0, 20.7=0,
         5.8=30, 6.8=100, 7.8=100, 8.8=30, 9.8=0, 10.8=0, 11.8=100, 12.8=30, 13.8=0, 14.8=70, 15.8=70, 16.8=70, 17.8=100, 18.8=100,
19.8=30, 20.8=0, 21.8=0,
         5.9=0, 6.9=30, 7.9=30, 8.9=30, 9.9=0, 10.9=30, 11.9=100, 12.9=30, 13.9=70, 14.9=70, 15.9=70, 16.9=0, 17.9=30, 18.9=100,
19.9=30, 20.9=30, 21.9=30,
         6.10=0, 7.10=30, 8.10=100, 9.10=30, 10.10=30, 11.10=30, 12.10=30, 13.10=70, 14.10=100, 15.10=100, 16.10=70, 17.10=0,
18.10=30, 19.10=30, 20.10=30, 21.10=100, 22.10=30,
         6.11=0, 7.11=30, 8.11=100, 9.11=30, 10.11=100, 11.11=30, 12.11=0, 13.11=0, 14.11=70, 15.11=70, 16.11=70, 17.11=0,
18.11=0, 19.11=0, 20.11=0, 21.11=30, 22.11=30,
         7.12=0, 8.12=30, 9.12=0, 10.12=30, 11.12=100, 12.12=30, 13.12=0, 14.12=70, 15.12=70, 16.12=30, 17.12=100, 18.12=30,
19.12=0, 20.12=0, 21.12=0, 22.12=0, 23.12=0,
         7.13=0, 8.13=30, 9.13=30, 10.13=0, 11.13=30, 12.13=100, 13.13=30, 14.13=70, 15.13=100, 16.13=70, 17.13=30, 18.13=30,
19.13=0, 20.13=0, 21.13=0, 22.13=0, 23.13=0,
         8.14=30, 9.14=100, 10.14=30, 11.14=0, 12.14=30, 13.14=30, 14.14=30, 15.14=0, 16.14=100, 17.14=70, 18.14=0, 19.14=0,
20.14=30, 21.14=30, 22.14=0, 23.14=0, 24.14=0,
         8.15=0, 9.15=30, 10.15=30, 11.15=0, 12.15=0, 13.15=30, 14.15=100, 15.15=30, 16.15=70, 17.15=70, 18.15=70, 19.15=0,
20.15=30, 21.15=100, 22.15=30, 23.15=0, 24.15=0,
         9.16=0, 10.16=30, 11.16=30, 12.16=0, 13.16=0, 14.16=30, 15.16=30, 16.16=0, 17.16=70, 18.16=100, 19.16=70, 20.16=0,
21.16=30, 22.16=30, 23.16=30, 24.16=30, 25.16=0,
         9.17=0, 10.17=30, 11.17=100, 12.17=30, 13.17=0, 14.17=0, 15.17=0, 16.17=30, 17.17=70, 18.17=70, 19.17=70, 20.17=0,
21.17=0, 22.17=0, 23.17=30, 24.17=100, 25.17=30,
         10.18=0, 11.18=30, 12.18=30, 13.18=0, 14.18=0, 15.18=0, 16.18=0, 17.18=0, 18.18=100, 19.18=70, 20.18=0, 21.18=0,
22.18=0, 23.18=0, 24.18=30, 25.18=30, 26.18=0,
         10.19=0, 11.19=0, 12.19=0, 13.19=0, 14.19=0, 15.19=0, 16.19=0, 17.19=0, 18.19=70, 19.19=70, 20.19=70, 21.19=0, 22.19=0,
23.19=0, 24.19=0, 25.19=0, 26.19=0,
         11.20=0, 12.20=0, 13.20=30, 14.20=30, 15.20=30, 16.20=0, 17.20=0, 18.20=0, 19.20=70, 20.20=100, 21.20=70, 22.20=0,
23.20=0, 24.20=30, 25.20=30, 26.20=0, 27.20=0,
         11.21=0, 12.21=0, 13.21=30, 14.21=100, 15.21=100, 16.21=30, 17.21=0, 18.21=70, 19.21=70, 20.21=70, 21.21=70, 22.21=0,
23.21=0, 24.21=30, 25.21=100, 26.21=30, 27.21=0,
         12.22=0, 13.22=0, 14.22=30, 15.22=30, 16.22=30, 17.22=0, 18.22=70, 19.22=100, 20.22=70, 21.22=0, 22.22=0, 23.22=0,
24.22=0, 25.22=30, 26.22=30, 27.22=30, 28.22=30,
         12.23=0, 13.23=0, 14.23=0, 15.23=0, 16.23=30, 17.23=30, 18.23=0, 19.23=70, 20.23=70, 21.23=0, 22.23=0, 23.23=30,
24.23=30, 25.23=0, 26.23=0, 27.23=30, 28.23=100,
```

```
        13.24=0,  14.24=0,  15.24=0,  16.24=30,  17.24=100,  18.24=30,  19.24=0,  20.24=0,  21.24=0,  22.24=0,  23.24=0,  24.24=100,
25.24=30, 26.24=0, 27.24=0, 28.24=30, 29.24=30,
        13.25=0, 14.25=0, 15.25=0, 16.25=0, 17.25=30, 18.25=30, 19.25=0, 20.25=0, 21.25=0, 22.25=0, 23.25=0, 24.25=30, 25.25=30,
26.25=0, 27.25=0, 28.25=0, 29.25=0/;
*================================== VARIABLES ================================
BINARY VARIABLE
        X(i,j) node equals 1 if a node is placed at grid i-j and 0 otherwise
        Y(i,j) zone equals 1 if no comms link to a node within range;

VARIABLE
        Z total uncovered importance of zones;

*================================== EQUATIONS ================================
EQUATIONS
        OBJ total uncovered importance
        WITHIN_RANGE(i,j)
        NO GO
        MAX_NODES;

*We seek to minimize the "total importance" of uncovered zones in obj function.
OBJ..
        Z =E= Sum( (i,j), Y(i,j)*importance(i,j) );

*For each zone, there must be at least a node at the zone, or at least one node
*at the 6 adjacent zones, or else Y(i,j) is set to 1, meaning that that
*a particular zone X(i,j) is not covered.
WITHIN_RANGE(i,j)..
          X(i-1,j-1) + X(i,j-1)
        + X(i-1,j) + X(i,j) + X(i+1,j)
        + X(i,j+1) + X(i+1,j+1)
        + Y(i,j) =G=1;

*These are zones that no node is allowed to be deployed
NO GO..
          X("8","1") + X("16","1")
        + X("4","2") + X("7","2")       + X("10","2")
        + X("12","3")
        + X("14","4") + X("17","4")     + X("9","4")
        + X("5","6")                    + X("14","6")
        + X("11","7") + X("12","7")
        + X("10","8") + X("19","8")     + X("11","8") + X("14","8")
                                        + X("14","10")
        + X("17","11")                  + X("10","11")
        + X("9","12")
        + X("15","14")                  + X("9","14")
        + X("20","15")                  + X("21","15")
                                        + X("18","16")
        + X("16","17")
        + X("16","18") + X("17","18")
        + X("23","19") + X("24","19")
                                        + X("15","21") + X("25","21")
        + X("23","24")
        + X("15","25") =E=0;

*Set the maximum number of nodes available for deployment
MAX_NODES..
        Sum( (i,j), X(i,j) ) =L=30;

*================================== MODEL ====================================
MODEL comms_gs5 /ALL/;
OPTION LP=OSL;
OPTION MIP=XA;
OPTION ITERLIM = 500000;
OPTION RESLIM = 100000;
SOLVE comms_gs5 USING MIP MINIMIZING Z;

*============================== DISPLAY RESULTS ==============================
DISPLAY
        Z.l
        X.l
        Y.l;
*==================================== END ====================================
```

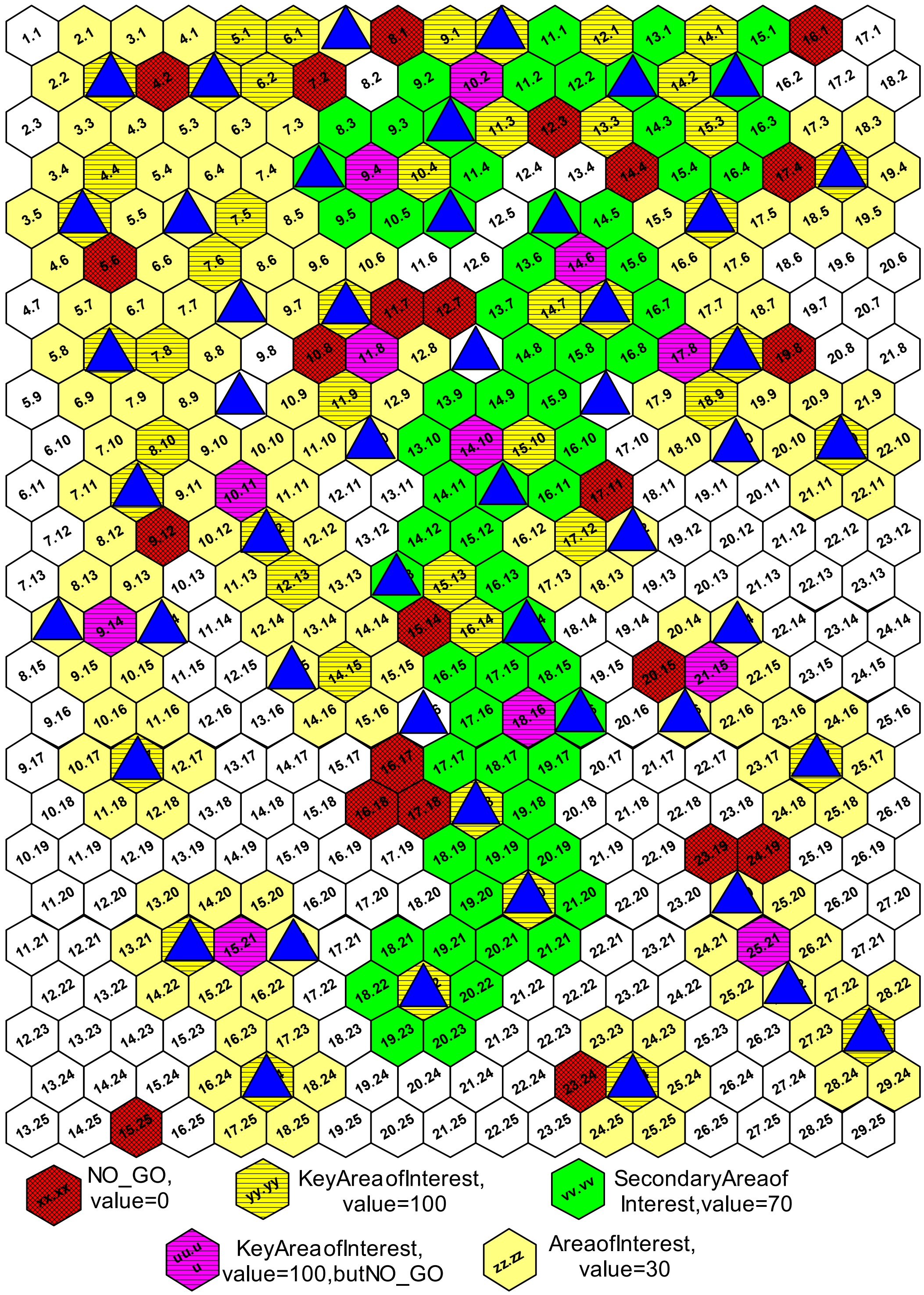

Figure 30. Node placement for Scenario Five.

GAMS Code for Scenario Six

```
*=========================== Generic scenario SIX =============================
*Filename: comms_gs6
*Using importance such that 100 to first 3 layers of areas of interest nearest
*to land, 90 to next 3 layers, 80 to next 3 layers,... and generic grid of 17x25.
*Comms range assumed to be 10km or 5.4NM
*To investigate the effects of having military values in a decreasing fashion
*as the distance from land increases.

*==================================== SETS ====================================
*(1,1) starts from the top left corner
SETS     I column grid number /1*29/
         J row grid number /1*25/;

*================================= PARAMETERS ================================
*Importance of zones, or can look at it as penalty if not covered
*Input values from 0(least important) to 100(most important),
*intermediate values are allowed.
PARAMETERS       importance(i,j) /
         1.1=0, 2.1=0, 3.1=0, 4.1=0, 5.1=100, 6.1=100, 7.1=0, 8.1=0, 9.1=100, 10.1=100, 11.1=0, 12.1=100, 13.1=0, 14.1=100, 15.1=0,
16.1=0, 17.1=0,
         2.2=0, 3.2=100, 4.2=0, 5.2=100, 6.2=100, 7.2=0, 8.2=0, 9.2=0, 10.2=100, 11.2=0, 12.2=0, 13.2=0, 14.2=100, 15.2=0, 16.2=0,
17.2=0, 18.2=0,
         2.3=0, 3.3=0, 4.3=0, 5.3=0, 6.3=0, 7.3=0, 8.3=0, 9.3=0, 10.3=0, 11.3=100, 12.3=0, 13.3=100, 14.3=0, 15.3=100, 16.3=0,
17.3=0, 18.3=0,
         3.4=0, 4.4=90, 5.4=0, 6.4=0, 7.4=0, 8.4=0, 9.4=90, 10.4=90, 11.4=0, 12.4=0, 13.4=0, 14.4=0, 15.4=0, 16.4=0, 17.4=0, 18.4=90,
19.4=0,
         3.5=0, 4.5=90, 5.5=0, 6.5=0, 7.5=90, 8.5=0, 9.5=0, 10.5=0, 11.5=0, 12.5=0, 13.5=0, 14.5=0, 15.5=0, 16.5=90, 17.5=0, 18.5=0,
19.5=0,
         4.6=0, 5.6=0, 6.6=0, 7.6=90, 8.6=0, 9.6=0, 10.6=0, 11.6=0, 12.6=0, 13.6=0, 14.6=90, 15.6=0, 16.6=0, 17.6=0, 18.6=0, 19.6=0,
20.6=0,
         4.7=0, 5.7=0, 6.7=0, 7.7=0, 8.7=0, 9.7=0, 10.7=80, 11.7=0, 12.7=0, 13.7=0, 14.7=80, 15.7=80, 16.7=0, 17.7=0, 18.7=0, 19.7=0,
20.7=0,
         5.8=0, 6.8=80, 7.8=80, 8.8=0, 9.8=0, 10.8=0, 11.8=80, 12.8=0, 13.8=0, 14.8=0, 15.8=0, 16.8=0, 17.8=80, 18.8=80, 19.8=0,
20.8=0, 21.8=0,
         5.9=0, 6.9=0, 7.9=0, 8.9=0, 9.9=0, 10.9=0, 11.9=80, 12.9=0, 13.9=0, 14.9=0, 15.9=0, 16.9=0, 17.9=0, 18.9=80, 19.9=0, 20.9=0,
21.9=0,
         6.10=0, 7.10=0, 8.10=70, 9.10=0, 10.10=0, 11.10=0, 12.10=0, 13.10=0, 14.10=70, 15.10=70, 16.10=0, 17.10=0, 18.10=0,
19.10=0, 20.10=0, 21.10=70, 22.10=0,
         6.11=0, 7.11=0, 8.11=70, 9.11=0, 10.11=70, 11.11=0, 12.11=0, 13.11=0, 14.11=0, 15.11=0, 16.11=0, 17.11=0, 18.11=0,
19.11=0, 20.11=0, 21.11=0, 22.11=0,
         7.12=0, 8.12=0, 9.12=0, 10.12=0, 11.12=70, 12.12=0, 13.12=0, 14.12=0, 15.12=0, 16.12=0, 17.12=70, 18.12=0, 19.12=0,
20.12=0, 21.12=0, 22.12=0, 23.12=0,
         7.13=0, 8.13=0, 9.13=0, 10.13=0, 11.13=0, 12.13=60, 13.13=0, 14.13=0, 15.13=60, 16.13=0, 17.13=0, 18.13=0, 19.13=0,
20.13=0, 21.13=0, 22.13=0, 23.13=0,
         8.14=0, 9.14=60, 10.14=0, 11.14=0, 12.14=0, 13.14=0, 14.14=0, 15.14=0, 16.14=60, 17.14=0, 18.14=0, 19.14=0, 20.14=0,
21.14=0, 22.14=0, 23.14=0, 24.14=0,
         8.15=0, 9.15=0, 10.15=0, 11.15=0, 12.15=0, 13.15=0, 14.15=60, 15.15=0, 16.15=0, 17.15=0, 18.15=0, 19.15=0, 20.15=0,
21.15=60, 22.15=0, 23.15=0, 24.15=0,
         9.16=0, 10.16=0, 11.16=0, 12.16=0, 13.16=0, 14.16=0, 15.16=0, 16.16=0, 17.16=0, 18.16=50, 19.16=0, 20.16=0, 21.16=0,
22.16=0, 23.16=0, 24.16=0, 25.16=0,
         9.17=0, 10.17=0, 11.17=50, 12.17=0, 13.17=0, 14.17=0, 15.17=0, 16.17=0, 17.17=0, 18.17=0, 19.17=0, 20.17=0, 21.17=0,
22.17=0, 23.17=0, 24.17=50, 25.17=0,
         10.18=0, 11.18=0, 12.18=0, 13.18=0, 14.18=0, 15.18=0, 16.18=0, 17.18=0, 18.18=50, 19.18=0, 20.18=0, 21.18=0, 22.18=0,
23.18=0, 24.18=0, 25.18=0, 26.18=0,
         10.19=0, 11.19=0, 12.19=0, 13.19=0, 14.19=0, 15.19=0, 16.19=0, 17.19=0, 18.19=0, 19.19=0, 20.19=0, 21.19=0, 22.19=0,
23.19=0, 24.19=0, 25.19=0, 26.19=0,
         11.20=0, 12.20=0, 13.20=0, 14.20=0, 15.20=0, 16.20=0, 17.20=0, 18.20=0, 19.20=0, 20.20=40, 21.20=0, 22.20=0, 23.20=0,
24.20=0, 25.20=0, 26.20=0, 27.20=0,
         11.21=0, 12.21=0, 13.21=0, 14.21=40, 15.21=40, 16.21=0, 17.21=0, 18.21=0, 19.21=0, 20.21=0, 21.21=0, 22.21=0, 23.21=0,
24.21=0, 25.21=40, 26.21=0, 27.21=0,
         12.22=0, 13.22=0, 14.22=0, 15.22=0, 16.22=0, 17.22=0, 18.22=0, 19.22=30, 20.22=0, 21.22=0, 22.22=0, 23.22=0, 24.22=0,
25.22=0, 26.22=0, 27.22=0, 28.22=0,
         12.23=0, 13.23=0, 14.23=0, 15.23=0, 16.23=0, 17.23=0, 18.23=0, 19.23=0, 20.23=0, 21.23=0, 22.23=0, 23.23=0, 24.23=0,
25.23=0, 26.23=0, 27.23=0, 28.23=30,
         13.24=0, 14.24=0, 15.24=0, 16.24=0, 17.24=30, 18.24=0, 19.24=0, 20.24=0, 21.24=0, 22.24=0, 23.24=0, 24.24=30, 25.24=0,
26.24=0, 27.24=0, 28.24=0, 29.24=0,
```

```
    13.25=0, 14.25=0, 15.25=0, 16.25=0, 17.25=0, 18.25=0, 19.25=0, 20.25=0, 21.25=0, 22.25=0, 23.25=0, 24.25=0, 25.25=0,
26.25=0, 27.25=0, 28.25=0, 29.25=0/;
*================================== VARIABLES =================================
BINARY VARIABLE
    X(i,j) node equals 1 if a node is placed at grid i-j and 0 otherwise
    Y(i,j) zone equals 1 if no comms link to a node within range;

VARIABLE
    Z total uncovered importance of zones;

*================================== EQUATIONS =================================
EQUATIONS
    OBJ total uncovered importance
    WITHIN_RANGE(i,j)
    NO GO
    MAX_NODES;

*We seek to minimize the "total importance" of uncovered zones in obj function.
OBJ..
    Z =E= Sum( (i,j), Y(i,j)*importance(i,j) );

*For each zone, there must be at least a node at the zone, or at least one node
*at the 6 adjacent zones, or else Y(i,j) is set to 1, meaning that that
*a particular zone X(i,j) is not covered.
WITHIN_RANGE(i,j)..
      X(i-1,j-1) + X(i,j-1)
    + X(i-1,j) + X(i,j) + X(i+1,j)
    + X(i,j+1) + X(i+1,j+1)
    + Y(i,j) =G=1;

*These are zones that no node is allowed to be deployed
NO GO..
      X("8","1") + X("16","1")
    + X("4","2") + X("7","2")
    + X("12","3")
    + X("14","4") + X("17","4")
    + X("5","6")
    + X("11","7") + X("12","7")
    + X("10","8") + X("19","8")
    + X("17","11")
    + X("9","12")
    + X("15","14")
    + X("20","15")
    + X("16","17")
    + X("16","18") + X("17","18")
    + X("23","19") + X("24","19")
    + X("23","24")
    + X("15","25") =E=0;

*Set the maximum number of nodes available for deployment
MAX_NODES..
    Sum( (i,j), X(i,j) ) =L=30;

*================================== MODEL =====================================
MODEL comms_gs6 /ALL/;
OPTION LP=OSL;
OPTION MIP=XA;
OPTION ITERLIM = 500000;
OPTION RESLIM = 100000;
SOLVE comms_gs6 USING MIP MINIMIZING Z;

*=============================== DISPLAY RESULTS ==============================
DISPLAY
    Z.l
    X.l
    Y.l;

*=================================== END ======================================
```

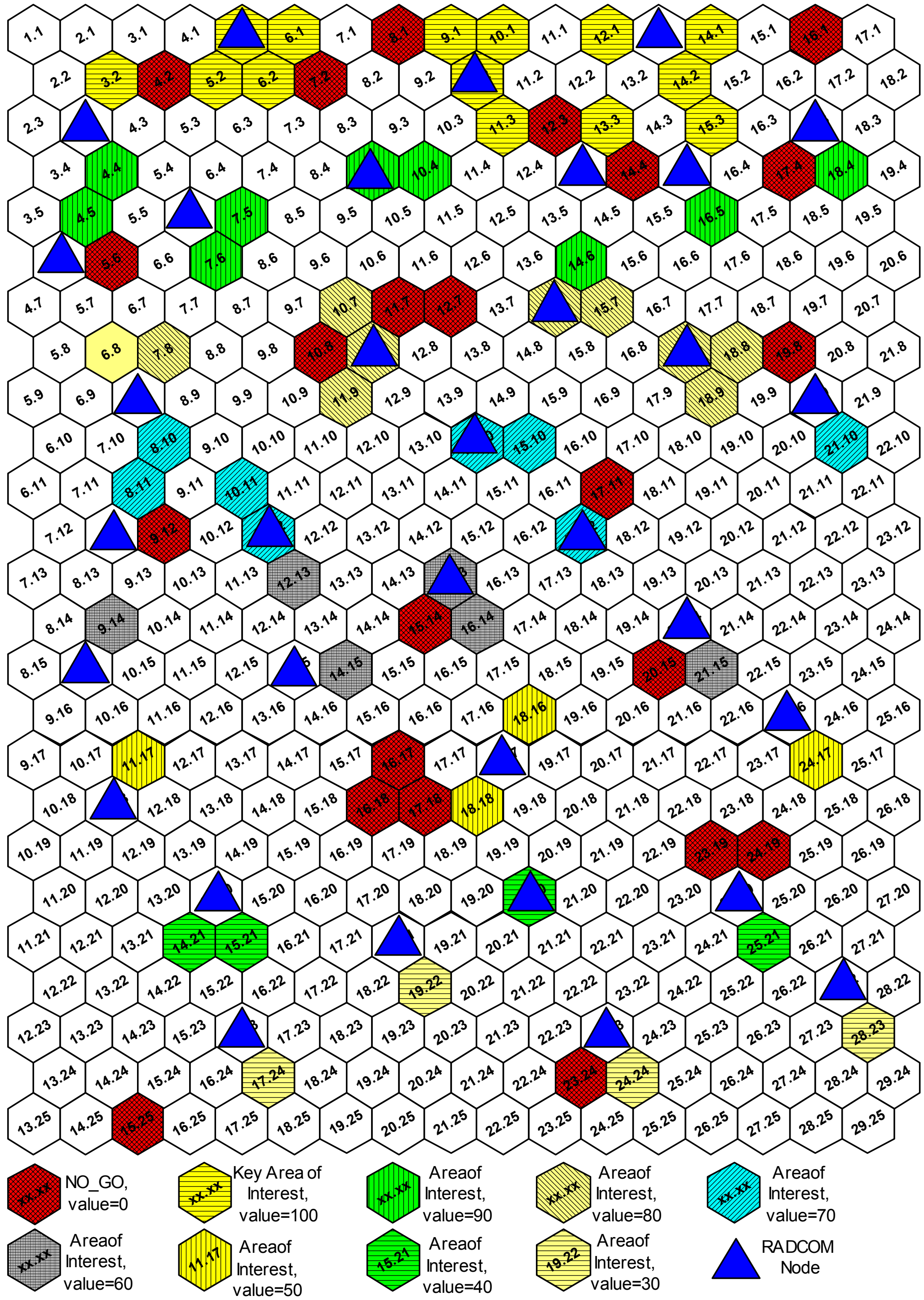

Figure 31. Node placement for Scenario Six.

GAMS Code for Scenario Seven

```
*========================== Generic scenario SEVEN ==========================
*Filename: comms_gs7
*Using importance with just either 0, 70(area of secondary interest)
*and 100 and generic grid of 17x25.
*Comms range assumed to be 2x 10km or 2x 5.4NM
*
*Investigate effects doubling the communication range capability.

*==================================== SETS ====================================
*(1,1) starts from the top left corner
SETS    I column grid number /1*29/
        J row grid number /1*25/;

*================================= PARAMETERS ================================
*Importance of zones, or can look at it as penalty if not covered
*Input values from 0(least important) to 100(most important),
*intermediate values are allowed.
PARAMETERS      importance(i,j) /
        1.1=0, 2.1=70, 3.1=70, 4.1=70, 5.1=100, 6.1=100, 7.1=70, 8.1=70, 9.1=100, 10.1=100, 11.1=70, 12.1=100, 13.1=70, 14.1=100,
15.1=70, 16.1=0, 17.1=0,
        2.2=70, 3.2=100, 4.2=70, 5.2=100, 6.2=100, 7.2=70, 8.2=0, 9.2=70, 10.2=100, 11.2=70, 12.2=70, 13.2=70, 14.2=100, 15.2=70,
16.2=0, 17.2=0, 18.2=0,
        2.3=0, 3.3=70, 4.3=70, 5.3=70, 6.3=70, 7.3=70, 8.3=70, 9.3=70, 10.3=70, 11.3=100, 12.3=0, 13.3=100, 14.3=70, 15.3=100,
16.3=70, 17.3=70, 18.3=70,
        3.4=70, 4.4=100, 5.4=70, 6.4=70, 7.4=70, 8.4=70, 9.4=100, 10.4=100, 11.4=70, 12.4=0, 13.4=0, 14.4=0, 15.4=70, 16.4=70,
17.4=0, 18.4=100, 19.4=70,
        3.5=70, 4.5=100, 5.5=70, 6.5=70, 7.5=100, 8.5=70, 9.5=70, 10.5=70, 11.5=70, 12.5=0, 13.5=70, 14.5=70, 15.5=70, 16.5=100,
17.5=70, 18.5=70, 19.5=70,
        4.6=70, 5.6=0, 6.6=70, 7.6=100, 8.6=70, 9.6=70, 10.6=70, 11.6=0, 12.6=0, 13.6=70, 14.6=100, 15.6=70, 16.6=70, 17.6=70,
18.6=0, 19.6=0, 20.6=0,
        4.7=0, 5.7=70, 6.7=70, 7.7=70, 8.7=70, 9.7=70, 10.7=100, 11.7=0, 12.7=0, 13.7=70, 14.7=100, 15.7=100, 16.7=70, 17.7=70,
18.7=70, 19.7=0, 20.7=0,
        5.8=70, 6.8=100, 7.8=100, 8.8=70, 9.8=0, 10.8=0, 11.8=100, 12.8=70, 13.8=0, 14.8=70, 15.8=70, 16.8=70, 17.8=100, 18.8=100,
19.8=70, 20.8=0, 21.8=0,
        5.9=0, 6.9=70, 7.9=70, 8.9=70, 9.9=0, 10.9=70, 11.9=100, 12.9=70, 13.9=70, 14.9=70, 15.9=70, 16.9=0, 17.9=70, 18.9=100,
19.9=70, 20.9=70, 21.9=70,
        6.10=0, 7.10=70, 8.10=100, 9.10=70, 10.10=70, 11.10=70, 12.10=70, 13.10=70, 14.10=100, 15.10=100, 16.10=70, 17.10=0,
18.10=70, 19.10=70, 20.10=70, 21.10=100, 22.10=70,
        6.11=0, 7.11=70, 8.11=100, 9.11=70, 10.11=100, 11.11=70, 12.11=0, 13.11=0, 14.11=70, 15.11=70, 16.11=70, 17.11=0,
18.11=0, 19.11=0, 20.11=0, 21.11=70, 22.11=70,
        7.12=0, 8.12=70, 9.12=0, 10.12=70, 11.12=100, 12.12=70, 13.12=0, 14.12=70, 15.12=70, 16.12=70, 17.12=100, 18.12=70,
19.12=0, 20.12=0, 21.12=0, 22.12=0, 23.12=0,
        7.13=0, 8.13=70, 9.13=70, 10.13=0, 11.13=70, 12.13=100, 13.13=70, 14.13=70, 15.13=100, 16.13=70, 17.13=70, 18.13=70,
19.13=0, 20.13=0, 21.13=0, 22.13=0, 23.13=0,
        8.14=70, 9.14=100, 10.14=70, 11.14=0, 12.14=70, 13.14=70, 14.14=70, 15.14=0, 16.14=100, 17.14=70, 18.14=0, 19.14=0,
20.14=70, 21.14=70, 22.14=0, 23.14=0, 24.14=0,
        8.15=0, 9.15=70, 10.15=70, 11.15=0, 12.15=0, 13.15=70, 14.15=100, 15.15=70, 16.15=70, 17.15=70, 18.15=70, 19.15=0,
20.15=70, 21.15=100, 22.15=70, 23.15=0, 24.15=0,
        9.16=0, 10.16=70, 11.16=70, 12.16=0, 13.16=0, 14.16=70, 15.16=70, 16.16=0, 17.16=70, 18.16=100, 19.16=70, 20.16=0,
21.16=70, 22.16=70, 23.16=70, 24.16=70, 25.16=0,
        9.17=0, 10.17=70, 11.17=100, 12.17=70, 13.17=0, 14.17=0, 15.17=0, 16.17=70, 17.17=70, 18.17=70, 19.17=70, 20.17=0,
21.17=0, 22.17=0, 23.17=70, 24.17=100, 25.17=70,
        10.18=0, 11.18=70, 12.18=70, 13.18=0, 14.18=0, 15.18=0, 16.18=0, 17.18=0, 18.18=100, 19.18=70, 20.18=0, 21.18=0,
22.18=0, 23.18=0, 24.18=70, 25.18=70, 26.18=0,
        10.19=0, 11.19=0, 12.19=0, 13.19=0, 14.19=0, 15.19=0, 16.19=0, 17.19=0, 18.19=70, 19.19=70, 20.19=70, 21.19=0, 22.19=0,
23.19=0, 24.19=0, 25.19=0, 26.19=0,
        11.20=0, 12.20=0, 13.20=70, 14.20=70, 15.20=70, 16.20=0, 17.20=0, 18.20=0, 19.20=70, 20.20=100, 21.20=70, 22.20=0,
23.20=0, 24.20=70, 25.20=70, 26.20=0, 27.20=0,
        11.21=0, 12.21=0, 13.21=70, 14.21=100, 15.21=100, 16.21=70, 17.21=0, 18.21=70, 19.21=70, 20.21=70, 21.21=70, 22.21=0,
23.21=0, 24.21=70, 25.21=100, 26.21=70, 27.21=0,
        12.22=0, 13.22=0, 14.22=70, 15.22=70, 16.22=70, 17.22=0, 18.22=70, 19.22=100, 20.22=70, 21.22=0, 22.22=0, 23.22=0,
24.22=0, 25.22=70, 26.22=70, 27.22=70, 28.22=70,
        12.23=0, 13.23=0, 14.23=0, 15.23=0, 16.23=70, 17.23=70, 18.23=0, 19.23=70, 20.23=70, 21.23=0, 22.23=0, 23.23=70,
24.23=70, 25.23=0, 26.23=0, 27.23=70, 28.23=100,
        13.24=0, 14.24=0, 15.24=0, 16.24=70, 17.24=100, 18.24=70, 19.24=0, 20.24=0, 21.24=0, 22.24=0, 23.24=0, 24.24=100,
25.24=70, 26.24=0, 27.24=0, 28.24=70, 29.24=70,
        13.25=0, 14.25=0, 15.25=0, 16.25=0, 17.25=70, 18.25=70, 19.25=0, 20.25=0, 21.25=0, 22.25=0, 23.25=0, 24.25=70, 25.25=70,
26.25=0, 27.25=0, 28.25=0, 29.25=0/;
```

```
*================================== VARIABLES ================================
BINARY VARIABLE
        X(i,j) node equals 1 if a node is placed at grid i-j and 0 otherwise
        Y(i,j) zone equals 1 if no comms link to a node within range;

VARIABLE
        Z total uncovered importance of zones;

*================================== EQUATIONS ==================================
EQUATIONS
        OBJ total uncovered importance
        WITHIN_RANGE(i,j)
        NO GO
        MAX_NODES;

*We seek to minimize the "total importance" of uncovered zones in obj function.
OBJ..
        Z =E= Sum( (i,j), Y(i,j)*importance(i,j) );

*For each zone, there must be at least a node at the zone, or at least one node
*at the 18 adjacent zones or else Y(i,j) is set to 1, meaning that that
*particular zone X(i,j) is not covered.
WITHIN_RANGE(i,j)..
          X(i-2,j-2) + X(i-1,j-2) + X(i,j-2)
        + X(i-2,j-1) + X(i-1,j-1) + X(i,j-1) + X(i+1,j-1)
        + X(i-2,j) + X(i-1,j) + X(i,j) + X(i+1,j) + X(i+2,j)
        + X(i-1,j+1) + X(i,j+1) + X(i+1,j+1) + X(i+2,j+1)
        + X(i,j+2) + X(i+1,j+2) + X(i+2,j+2)
        +  Y(i,j) =G=1;
*these are zones that no node is allowed to be deployed
NO GO..
          X("8","1") + X("16","1")
        + X("4","2") + X("7","2")
        + X("12","3")
        + X("14","4") + X("17","4")
        + X("5","6")
        + X("11","7") + X("12","7")
        + X("10","8") + X("19","8")

        + X("17","11")
        + X("9","12")
        + X("15","14")
        + X("20","15")

        + X("16","17")
        + X("16","18") + X("17","18")
        + X("23","19") + X("24","19")

        + X("23","24")
        + X("15","25") =E=0;

*Set the maximum number of nodes available for deployment
MAX_NODES..
        Sum( (i,j), X(i,j) ) =L=22;

*================================== MODEL ====================================
MODEL comms_gs7 /ALL/;
OPTION LP=OSL;
OPTION MIP=XA;
OPTION ITERLIM = 500000;
OPTION RESLIM = 100000;
SOLVE comms_gs7 USING MIP MINIMIZING Z;

*============================== DISPLAY RESULTS ==============================
DISPLAY
        Z.l
        X.l
        Y.l;

*==================================== END ====================================
```

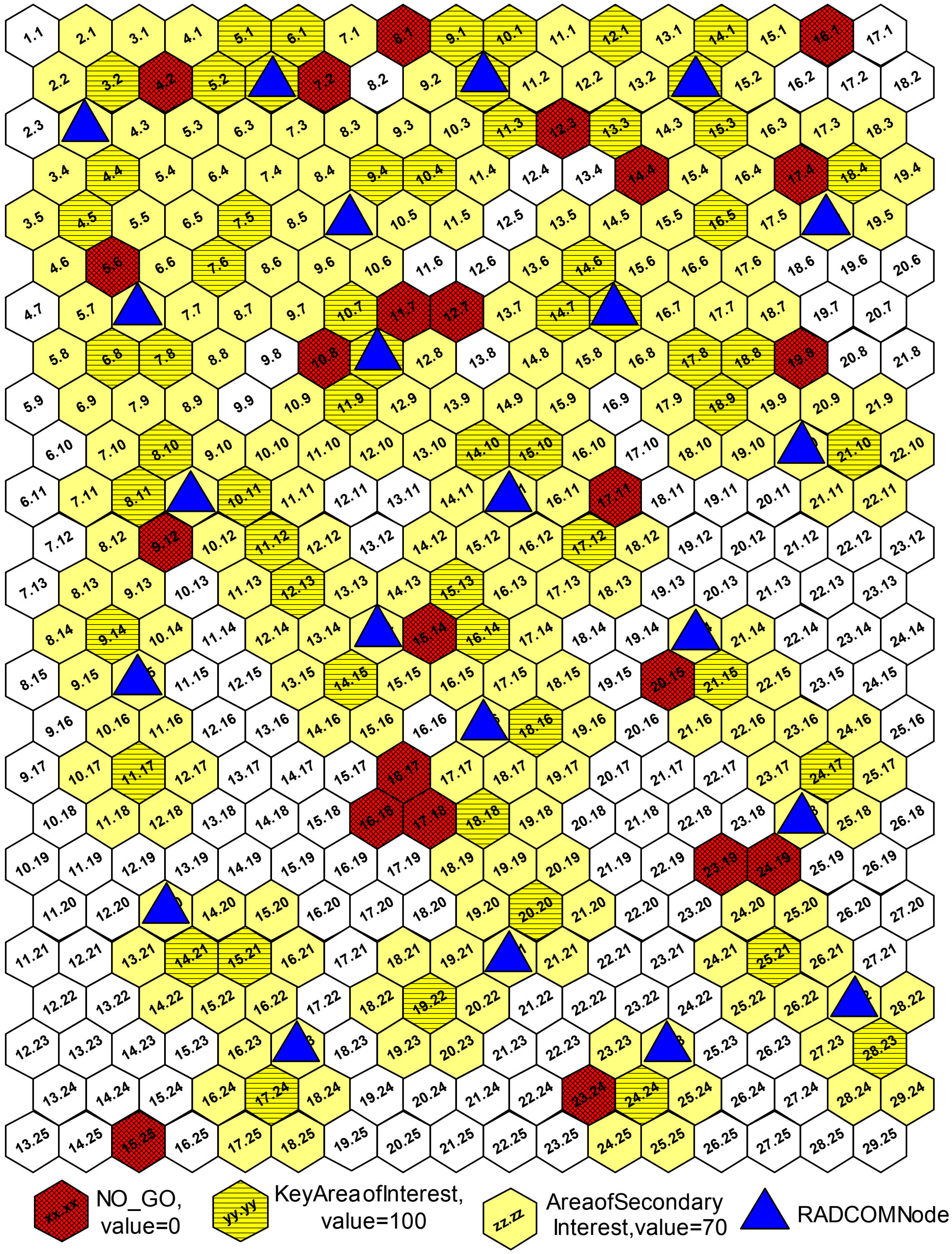

Figure 32. Node placement for Scenario Seven.

GAMS Code for Integrated Project Alpha

```
*================= Integrated Project specific scenario ALPHA ==================
*Filename: comms_ip1
*Using importance with just 0, 30(area of secondary interest), 100 (key areas of
*interest) from Integrated Project Team requirement and AO of 80NM x 100NM.
*Comms range assumed to be 10km or 5.4NM

*==================================== SETS ====================================
*(1,1) starts from the top left corner
SETS     I column grid number /1*31/
         J row grid number /1*26/;

*================================== PARAMETERS ================================
*Importance of zones, or can look at it as penalty if not covered
*Input values from 0(least important or land in some cases) to 100(most
*important), intermediate values are allowed.
*30 represent areas of secondary importance. Assumed that NO GO areas still
*have importance, just simply imposed the constraint of no deployment but
*coverge is still required.
PARAMETERS       importance(i,j) /
         1.1=100, 2.1=0, 3.1=0, 4.1=100, 5.1=100, 6.1=0, 7.1=0, 8.1=100, 9.1=0, 10.1=0, 11.1=0, 12.1=0, 13.1=0, 14.1=0, 15.1=0, 16.1=0, 17.1=0,
         2.2=100, 3.2=0, 4.2=100, 5.2=100, 6.2=100, 7.2=0, 8.2=100, 9.2=100, 10.2=0, 11.2=0, 12.2=0, 13.2=0, 14.2=100, 15.2=100, 16.2=100, 17.2=100, 18.2=100,
         2.3=100, 3.3=100, 4.3=100, 5.3=100, 6.3=100, 7.3=100, 8.3=100, 9.3=100, 10.3=100, 11.3=0, 12.3=0, 13.3=0, 14.3=100, 15.3=100, 16.3=100, 17.3=100, 18.3=100,
         3.4=100, 4.4=100, 5.4=100, 6.4=100, 7.4=100, 8.4=100, 9.4=100, 10.4=100, 11.4=100, 12.4=100, 13.4=100, 14.4=100, 15.4=100, 16.4=100, 17.4=100, 18.4=100, 19.4=100,
         3.5=100, 4.5=100, 5.5=100, 6.5=100, 7.5=100, 8.5=100, 9.5=100, 10.5=100, 11.5=100, 12.5=100, 13.5=100, 14.5=100, 15.5=100, 16.5=100, 17.5=100, 18.5=100, 19.5=100,
         4.6=100, 5.6=100, 6.6=100, 7.6=100, 8.6=100, 9.6=100, 10.6=100, 11.6=100, 12.6=100, 13.6=100, 14.6=100, 15.6=100, 16.6=100, 17.6=100, 18.6=100, 19.6=100, 20.6=100,
         4.7=100, 5.7=100, 6.7=100, 7.7=100, 8.7=100, 9.7=100, 10.7=100, 11.7=100, 12.7=100, 13.7=100, 14.7=100, 15.7=100, 16.7=100, 17.7=100, 18.7=100, 19.7=100, 20.7=100,
         5.8=100, 6.8=100, 7.8=100, 8.8=100, 9.8=100, 10.8=100, 11.8=100, 12.8=100, 13.8=100, 14.8=100, 15.8=100, 16.8=100, 17.8=100, 18.8=100, 19.8=100, 20.8=100, 21.8=100,
         5.9=100, 6.9=100, 7.9=100, 8.9=100, 9.9=100, 10.9=100, 11.9=100, 12.9=100, 13.9=100, 14.9=100, 15.9=100, 16.9=100, 17.9=100, 18.9=100, 19.9=100, 20.9=100, 21.9=100,
         6.10=30, 7.10=30, 8.10=30, 9.10=30, 10.10=30, 11.10=30, 12.10=30, 13.10=30, 14.10=30, 15.10=30, 16.10=30, 17.10=30, 18.10=30, 19.10=30, 20.10=30, 21.10=30, 22.10=30,
         6.11=30, 7.11=30, 8.11=30, 9.11=30, 10.11=30, 11.11=30, 12.11=30, 13.11=30, 14.11=30, 15.11=30, 16.11=30, 17.11=30, 18.11=30, 19.11=30, 20.11=30, 21.11=30, 22.11=30,
         7.12=30, 8.12=30, 9.12=30, 10.12=30, 11.12=30, 12.12=30, 13.12=30, 14.12=30, 15.12=30, 16.12=30, 17.12=30, 18.12=30, 19.12=30, 20.12=30, 21.12=30, 22.12=30, 23.12=30,
         7.13=30, 8.13=30, 9.13=30, 10.13=30, 11.13=30, 12.13=30, 13.13=30, 14.13=30, 15.13=30, 16.13=30, 17.13=30, 18.13=30, 19.13=30, 20.13=30, 21.13=30, 22.13=30, 23.13=30,
         8.14=30, 9.14=30, 10.14=30, 11.14=30, 12.14=30, 13.14=30, 14.14=30, 15.14=30, 16.14=30, 17.14=30, 18.14=30, 19.14=30, 20.14=30, 21.14=30, 22.14=30, 23.14=30, 24.14=30,
         8.15=30, 9.15=30, 10.15=30, 11.15=30, 12.15=30, 13.15=30, 14.15=30, 15.15=30, 16.15=30, 17.15=30, 18.15=30, 19.15=30, 20.15=30, 21.15=30, 22.15=30, 23.15=30, 24.15=30,
         9.16=30, 10.16=30, 11.16=30, 12.16=30, 13.16=30, 14.16=30, 15.16=30, 16.16=30, 17.16=30, 18.16=30, 19.16=30, 20.16=30, 21.16=30, 22.16=30, 23.16=30, 24.16=30, 25.16=30,
         9.17=30, 10.17=30, 11.17=30, 12.17=30, 13.17=30, 14.17=30, 15.17=30, 16.17=30, 17.17=30, 18.17=30, 19.17=30, 20.17=30, 21.17=30, 22.17=30, 23.17=30, 24.17=30, 25.17=30,
         10.18=30, 11.18=30, 12.18=30, 13.18=30, 14.18=30, 15.18=30, 16.18=30, 17.18=30, 18.18=30, 19.18=30, 20.18=30, 21.18=30, 22.18=30, 23.18=30, 24.18=30, 25.18=30, 26.18=30,
         10.19=30, 11.19=30, 12.19=30, 13.19=30, 14.19=30, 15.19=30, 16.19=30, 17.19=30, 18.19=30, 19.19=30, 20.19=30, 21.19=30, 22.19=30, 23.19=30, 24.19=30, 25.19=30, 26.19=30,
         11.20=30, 12.20=30, 13.20=30, 14.20=30, 15.20=30, 16.20=30, 17.20=30, 18.20=30, 19.20=30, 20.20=30, 21.20=30, 22.20=30, 23.20=30, 24.20=30, 25.20=30, 26.20=30, 27.20=30,
         11.21=30, 12.21=30, 13.21=30, 14.21=30, 15.21=30, 16.21=30, 17.21=30, 18.21=30, 19.21=30, 20.21=30, 21.21=30, 22.21=30, 23.21=30, 24.21=30, 25.21=30, 26.21=30, 27.21=30,
         12.22=30, 13.22=30, 14.22=30, 15.22=30, 16.22=30, 17.22=30, 18.22=30, 19.22=30, 20.22=30, 21.22=30, 22.22=30, 23.22=30, 24.22=30, 25.22=30, 26.22=30, 27.22=30, 28.22=30,
         12.23=30, 13.23=30, 14.23=30, 15.23=30, 16.23=30, 17.23=30, 18.23=30, 19.23=30, 20.23=30, 21.23=30, 22.23=30, 23.23=30, 24.23=30, 25.23=30, 26.23=30, 27.23=30, 28.23=30,
         13.24=30, 14.24=30, 15.24=30, 16.24=30, 17.24=30, 18.24=30, 19.24=30, 20.24=30, 21.24=30, 22.24=30, 23.24=30, 24.24=30, 25.24=30, 26.24=30, 27.24=30, 28.24=30, 29.24=30,
```

```
        13.25=30, 14.25=30, 15.25=30, 16.25=30, 17.25=30, 18.25=30, 19.25=30, 20.25=30, 21.25=30, 22.25=30, 23.25=30, 24.25=30,
25.25=30, 26.25=30, 27.25=30, 28.25=30, 29.25=30/;

*=================================== VARIABLES ==================================
BINARY VARIABLE
        X(i,j) node equals 1 if a node is placed at grid i-j and 0 otherwise
        Y(i,j) zone equals 1 if no comms link to a node within range;

VARIABLE
        Z total uncovered importance of zones;

*=================================== EQUATIONS ==================================
EQUATIONS
        OBJ total uncovered importance
        WITHIN_RANGE(i,j)
        NO GO
        BOUNDARY
        MAX_NODES;

*We seek to minimize the "total importance" of uncovered zones in obj function.
OBJ..
        Z =E= Sum( (i,j), Y(i,j)*importance(i,j) );

*For each zone, there must be at least a node at the zone, or at least one node
*at the 6 adjacent zones, or else Y(i,j) is set to 1, meaning that that
*a particular zone X(i,j) is not covered.
WITHIN_RANGE(i,j)..
          X(i-1,j-1) + X(i,j-1)
        + X(i-1,j) + X(i,j) + X(i+1,j)
        + X(i,j+1) + X(i+1,j+1)
        +  Y(i,j) =G=1;

*These are zones that no node is allowed to be deployed
NO GO..
          X("2","1") + X("3","1") + X("6","1") + X("7","1") + X("9","1") + X("10","1") + X("11","1")
        + X("12","1") + X("13","1") + X("14","1") + X("15","1") + X("16","1") + X("17","1")
        + X("3","2") + X("7","2") + X("10","2") + X("11","2")+ X("12","2") + X("13","2") + X("14","2")
        + X("11","3") + X("12","3") + X("13","3")
        + X("4","4") + X("8","4") + X("18","4")
        + X("12","5") + X("13","5") + X("16","5")
        + X("4","6") + X("8","6") + X("10","6") + X("12","6")
        + X("16","7") + X("18","7")
        + X("19","8")
        + X("15","9")
        + X("13","10")
        + X("11","11")
        + X("11","14")
        + X("17","15")
        + X("11","17")+ X("21","17") + X("24","17")
        + X("14","18")
        + X("18","19")
        + X("15","22") =E=0;

*To set the boundary of the area of operation, thus setting the boundary
*where no RADCOM nodes are allow to be deployed.
BOUNDARY..
*the following form the western boundary
          X("1","2") + X("2","2") + X("2","4") + X("2","5") + X("3","6") + X("3","7") + X("4","8")
        + X("4","9") + X("5","10") + X("5","11") + X("6","12") + X("6","13") + X("7","14")
        + X("7","15") + X("8","16") + X("8","17") + X("9","18")+ X("9","19") + X("10","20") + X("10","21")
        + X("11","22") + X("11","23") + X("12","24") + X("12","25")
*the following form the southern boundary
        + X("13","26") + X("14","26") + X("15","26") + X("16","26") + X("17","26") + X("18","26") + X("19","26")
        + X("20","26") + X("21","26") + X("22","26") + X("23","26") + X("24","26") + X("25","26")
        + X("26","26") + X("27","26") + X("28","26") + X("29","26") + X("30","26") + X("31","26")
*the following form the eastern boundary
        + X("19","2") + X("19","3") + X("20","4") + X("20","5") + X("21","6") + X("21","7") + X("22","8")
        + X("13","26") + X("13","26") + X("13","26") + X("13","26") + X("13","26") + X("13","26") + X("13","26")
        + X("22","9") + X("23","10") + X("23","11") + X("24","12") + X("24","13") + X("25","14") + X("25","15")
        + X("26","16") + X("26","17") + X("27","18") + X("27","19") + X("28","20") + X("28","21") + X("29","22")
```

```
        + X("29","23") + X("30","24") + X("30","25")
*the following form the northern boundary (shoreline)
        + X("2","1") + X("3","1")  + X("6","1") + X("7","1") + X("8","1") + X("9","1") + X("10","1") + X("11","1")
        + X("12","1") + X("13","1") + X("14","1") + X("15","1") + X("16","1") + X("17","1")
        + X("3","2") + X("7","2") + X("10","2") + X("11","2")+ X("12","2") + X("13","2") + X("16","2")
        + X("11","3") + X("12","3") + X("13","3")
        =E=0;

*Set the maximum number of nodes available for deployment
MAX_NODES..
        Sum( (i,j), X(i,j) ) =L=30;

*==================================== MODEL =======================================
MODEL comms_ip1 /ALL/;
OPTION LP=OSL;
OPTION MIP=XA;
OPTION ITERLIM = 500000;
OPTION RESLIM = 100000;
SOLVE comms_ip1 USING MIP MINIMIZING Z;

*================================ DISPLAY RESULTS ================================
DISPLAY
        Z.l
        X.l
        Y.l;

*===================================== END ==========================================
```

GAMS Code for Integrated Project Scenario Bravo

```
*================= Integrated Project specific scenario BRAVO ==================
*Filename: comms_ip2
*Using importance with just 0, 30(area of secondary interest), 100 (key areas of
*interest) from Integrated Project team requirement and AO of 80NM x 100NM.
*Comms range assumed to be 2x 10km or 2x 5.4NM
*To investigate the effects of doubling the communication range.

*==================================== SETS ====================================
*(1,1) starts from the top left corner
SETS    I column grid number /1*31/
        J row grid number /1*26/;

*================================== PARAMETERS ================================
*Importance of zones, or can look at it as penalty if not covered
*Input values from 0(least important or land in some cases) to 100(most
*important), intermediate values are allowed.
*30 represent areas of secondary importance. Assumed that NO GO areas still
*have importance, just simply imposed the constraint of no deployment but
*coverge is still required.
PARAMETERS      importance(i,j) /
        1.1=100, 2.1=0, 3.1=0, 4.1=100, 5.1=100, 6.1=0, 7.1=0, 8.1=100, 9.1=0, 10.1=0, 11.1=0, 12.1=0, 13.1=0, 14.1=0, 15.1=0,
16.1=0, 17.1=0,
        2.2=100, 3.2=0, 4.2=100, 5.2=100, 6.2=100, 7.2=0, 8.2=100, 9.2=100, 10.2=0, 11.2=0, 12.2=0, 13.2=0, 14.2=100, 15.2=100,
16.2=100, 17.2=100, 18.2=100,
        2.3=100, 3.3=100, 4.3=100, 5.3=100, 6.3=100, 7.3=100, 8.3=100, 9.3=100, 10.3=100, 11.3=0, 12.3=0, 13.3=0, 14.3=100,
15.3=100, 16.3=100, 17.3=100, 18.3=100,
        3.4=100, 4.4=100, 5.4=100, 6.4=100, 7.4=100, 8.4=100, 9.4=100, 10.4=100, 11.4=100, 12.4=100, 13.4=100, 14.4=100,
15.4=100, 16.4=100, 17.4=100, 18.4=100, 19.4=100,
        3.5=100, 4.5=100, 5.5=100, 6.5=100, 7.5=100, 8.5=100, 9.5=100, 10.5=100, 11.5=100, 12.5=100, 13.5=100, 14.5=100,
15.5=100, 16.5=100, 17.5=100, 18.5=100, 19.5=100,
        4.6=100, 5.6=100, 6.6=100, 7.6=100, 8.6=100, 9.6=100, 10.6=100, 11.6=100, 12.6=100, 13.6=100, 14.6=100, 15.6=100,
16.6=100, 17.6=100, 18.6=100, 19.6=100, 20.6=100,
        4.7=100, 5.7=100, 6.7=100, 7.7=100, 8.7=100, 9.7=100, 10.7=100, 11.7=100, 12.7=100, 13.7=100, 14.7=100, 15.7=100,
16.7=100, 17.7=100, 18.7=100, 19.7=100, 20.7=100,
        5.8=100, 6.8=100, 7.8=100, 8.8=100, 9.8=100, 10.8=100, 11.8=100, 12.8=100, 13.8=100, 14.8=100, 15.8=100, 16.8=100,
17.8=100, 18.8=100, 19.8=100, 20.8=100, 21.8=100,
        5.9=100, 6.9=100, 7.9=100, 8.9=100, 9.9=100, 10.9=100, 11.9=100, 12.9=100, 13.9=100, 14.9=100, 15.9=100, 16.9=100,
17.9=100, 18.9=100, 19.9=100, 20.9=100, 21.9=100,
        6.10=30, 7.10=30, 8.10=30, 9.10=30, 10.10=30, 11.10=30, 12.10=30, 13.10=30, 14.10=30, 15.10=30, 16.10=30, 17.10=30,
18.10=30, 19.10=30, 20.10=30, 21.10=30, 22.10=30,
        6.11=30, 7.11=30, 8.11=30, 9.11=30, 10.11=30, 11.11=30, 12.11=30, 13.11=30, 14.11=30, 15.11=30, 16.11=30, 17.11=30,
18.11=30, 19.11=30, 20.11=30, 21.11=30, 22.11=30,
        7.12=30, 8.12=30, 9.12=30, 10.12=30, 11.12=30, 12.12=30, 13.12=30, 14.12=30, 15.12=30, 16.12=30, 17.12=30, 18.12=30,
19.12=30, 20.12=30, 21.12=30, 22.12=30, 23.12=30,
        7.13=30, 8.13=30, 9.13=30, 10.13=30, 11.13=30, 12.13=30, 13.13=30, 14.13=30, 15.13=30, 16.13=30, 17.13=30, 18.13=30,
19.13=30, 20.13=30, 21.13=30, 22.13=30, 23.13=30,
        8.14=30, 9.14=30, 10.14=30, 11.14=30, 12.14=30, 13.14=30, 14.14=30, 15.14=30, 16.14=30, 17.14=30, 18.14=30, 19.14=30,
20.14=30, 21.14=30, 22.14=30, 23.14=30, 24.14=30,
        8.15=30, 9.15=30, 10.15=30, 11.15=30, 12.15=30, 13.15=30, 14.15=30, 15.15=30, 16.15=30, 17.15=30, 18.15=30, 19.15=30,
20.15=30, 21.15=30, 22.15=30, 23.15=30, 24.15=30,
        9.16=30, 10.16=30, 11.16=30, 12.16=30, 13.16=30, 14.16=30, 15.16=30, 16.16=30, 17.16=30, 18.16=30, 19.16=30, 20.16=30,
21.16=30, 22.16=30, 23.16=30, 24.16=30, 25.16=30,
        9.17=30, 10.17=30, 11.17=30, 12.17=30, 13.17=30, 14.17=30, 15.17=30, 16.17=30, 17.17=30, 18.17=30, 19.17=30, 20.17=30,
21.17=30, 22.17=30, 23.17=30, 24.17=30, 25.17=30,
        10.18=30, 11.18=30, 12.18=30, 13.18=30, 14.18=30, 15.18=30, 16.18=30, 17.18=30, 18.18=30, 19.18=30, 20.18=30, 21.18=30,
22.18=30, 23.18=30, 24.18=30, 25.18=30, 26.18=30,
        10.19=30, 11.19=30, 12.19=30, 13.19=30, 14.19=30, 15.19=30, 16.19=30, 17.19=30, 18.19=30, 19.19=30, 20.19=30, 21.19=30,
22.19=30, 23.19=30, 24.19=30, 25.19=30, 26.19=30,
        11.20=30, 12.20=30, 13.20=30, 14.20=30, 15.20=30, 16.20=30, 17.20=30, 18.20=30, 19.20=30, 20.20=30, 21.20=30, 22.20=30,
23.20=30, 24.20=30, 25.20=30, 26.20=30, 27.20=30,
        11.21=30, 12.21=30, 13.21=30, 14.21=30, 15.21=30, 16.21=30, 17.21=30, 18.21=30, 19.21=30, 20.21=30, 21.21=30, 22.21=30,
23.21=30, 24.21=30, 25.21=30, 26.21=30, 27.21=30,
        12.22=30, 13.22=30, 14.22=30, 15.22=30, 16.22=30, 17.22=30, 18.22=30, 19.22=30, 20.22=30, 21.22=30, 22.22=30, 23.22=30,
24.22=30, 25.22=30, 26.22=30, 27.22=30, 28.22=30,
        12.23=30, 13.23=30, 14.23=30, 15.23=30, 16.23=30, 17.23=30, 18.23=30, 19.23=30, 20.23=30, 21.23=30, 22.23=30, 23.23=30,
24.23=30, 25.23=30, 26.23=30, 27.23=30, 28.23=30,
```

```
        13.24=30, 14.24=30, 15.24=30, 16.24=30, 17.24=30, 18.24=30, 19.24=30, 20.24=30, 21.24=30, 22.24=30, 23.24=30, 24.24=30,
25.24=30, 26.24=30, 27.24=30, 28.24=30, 29.24=30,
        13.25=30, 14.25=30, 15.25=30, 16.25=30, 17.25=30, 18.25=30, 19.25=30, 20.25=30, 21.25=30, 22.25=30, 23.25=30, 24.25=30,
25.25=30, 26.25=30, 27.25=30, 28.25=30, 29.25=30/;

*=================================== VARIABLES ==================================
BINARY VARIABLE
        X(i,j) node equals 1 if a node is placed at grid i-j and 0 otherwise
        Y(i,j) zone equals 1 if no comms link to a node within range;

VARIABLE
        Z total uncovered importance of zones;

*=================================== EQUATIONS ==================================
EQUATIONS
        OBJ total uncovered importance
        WITHIN_RANGE(i,j)
        NO GO
        BOUNDARY
        MAX_NODES;

*We seek to minimize the "total importance" of uncovered zones in obj function.
OBJ..
        Z =E= Sum( (i,j), Y(i,j)*importance(i,j) );

*For each zone, there must be at least a node at the zone, or at least one node
*at the 18 adjacent zones, or else Y(i,j) is set to 1, meaning that that
*a particular zone X(i,j) is not covered.
WITHIN_RANGE(i,j)..
          X(i-2,j-2) + X(i-1,j-2) + X(i,j-2)
        + X(i-2,j-1) + X(i-1,j-1) + X(i,j-1) + X(i+1,j-1)
        + X(i-2,j) + X(i-1,j) + X(i,j) + X(i+1,j) + X(i+2,j)
        + X(i-1,j+1) + X(i,j+1) + X(i+1,j+1) + X(i+2,j+1)
        + X(i,j+2) + X(i+1,j+2) + X(i+2,j+2)
        +  Y(i,j) =G=1;

*These are zones that no node is allowed to be deployed
NO GO..
          X("4","4") + X("8","4") + X("18","4")
        + X("12","5") + X("13","5") + X("16","5")
        + X("4","6") + X("8","6") + X("10","6") + X("12","6")
        + X("16","7") + X("18","7")
        + X("19","8")
        + X("15","9")
        + X("13","10")
        + X("11","11")
        + X("11","14")
        + X("17","15")
        + X("11","17")+ X("21","17") + X("24","17")
        + X("14","18")
        + X("18","19")
        + X("15","22") =E=0;

*To set the boundary of the area of operation, thus setting the boundary
*where no RADCOM nodes are allow to be deployed.
BOUNDARY..
*the following form the western boundary
          X("1","2") + X("2","2") + X("2","4") + X("2","5") + X("3","6") + X("3","7") + X("4","8")
        + X("4","9") + X("5","10") + X("5","11") + X("6","12") + X("6","13") + X("7","14")
        + X("7","15") + X("8","16") + X("8","17") + X("9","18")+ X("9","19") + X("10","20") + X("10","21")
        + X("11","22") + X("11","23") + X("12","24") + X("12","25")
*the following form the southern boundary
        + X("13","26") + X("14","26") + X("15","26") + X("16","26") + X("17","26") + X("18","26") + X("19","26")
        + X("20","26") + X("21","26") + X("22","26") + X("23","26") + X("24","26") + X("25","26")
        + X("26","26") + X("27","26") + X("28","26") + X("29","26") + X("30","26") + X("31","26")
*the following form the eastern boundary
        + X("19","2") + X("19","3") + X("20","4") + X("20","5") + X("21","6") + X("21","7") + X("22","8")
        + X("13","26") + X("13","26") + X("13","26") + X("13","26") + X("13","26") + X("13","26") + X("13","26")
        + X("22","9") + X("23","10") + X("23","11") + X("24","12") + X("24","13") + X("25","14") + X("25","15")
        + X("26","16") + X("26","17") + X("27","18") + X("27","19") + X("28","20") + X("28","21") + X("29","22")
```

```
        + X("29","23") + X("30","24") + X("30","25")
*the following form the northern boundary (shoreline)
        + X("2","1") + X("3","1")  + X("6","1") + X("7","1") + X("8","1") + X("9","1") + X("10","1") + X("11","1")
        + X("12","1") + X("13","1") + X("14","1") + X("15","1") + X("16","1") + X("17","1")
        + X("3","2") + X("7","2") + X("10","2") + X("11","2")+ X("12","2") + X("13","2") + X("16","2")
        + X("11","3") + X("12","3") + X("13","3")
        =E=0;

*Set the maximum number of nodes available for deployment
MAX_NODES..
        Sum( (i,j), X(i,j) ) =L=30;

*=================================== MODEL ======================================
MODEL comms_ip2 /ALL/;
OPTION LP=OSL;
OPTION MIP=XA;
OPTION ITERLIM = 500000;
OPTION RESLIM = 100000;
SOLVE comms_ip2 USING MIP MINIMIZING Z;

*=============================== DISPLAY RESULTS ==============================
DISPLAY
        Z.l
        X.l
        Y.l;

*==================================== END =======================================
```

APPENDIX III.A

Justification for Focused Search

FLIR EO/IR Ranges are a function of Field of View (FOV).
In order to see this relationship, first consider that the average screen consists of 1,024 pixels on the horizontal and 768 pixels on the vertical. Next, we calculate the possible view section width at various fields of view and slant range where: Possible View Section Width = Slant Range * Sin(FOV).

		Possible View Section Width						
	FOV	Slant Range (NM)						
FOV (Degrees)	(Radians)	0.25	0.5	1	3	6	12	20
1.5	0.0262	0.0065	0.0131	0.0262	0.0785	0.1571	0.3141	0.5235
5	0.0873	0.0218	0.0436	0.0872	0.2615	0.5229	1.0459	1.7431
15	0.2618	0.0647	0.1294	0.2588	0.7765	1.5529	3.1058	5.1764
22.5	0.3927	0.0957	0.1913	0.3827	1.1481	2.2961	4.5922	7.6537
30	0.5236	0.1250	0.2500	0.5000	1.5000	3.0000	6.0000	10.0000
45	0.7854	0.1768	0.3536	0.7071	2.1213	4.2426	8.4853	14.1421
60	1.0472	0.2165	0.4330	0.8660	2.5981	5.1962	10.3923	17.3205

Next, if we take each Possible View Section Width, convert to yards (X 2,000 yds/NM), and divide by the 1,024 pixel sceenwidth, we create the following chart for Horizontal Range Resolution:

		Horizontal Range Resolution (yds/pixel)						
	FOV	Slant Range (NM)						
FOV (Degrees)	(Radians)	0.25	0.5	1	3	6	12	20
1.5	0.0262	0.0128	0.0256	0.0511	0.1534	0.3068	0.6135	1.0225
5	0.0873	0.0426	0.0851	0.1702	0.5107	1.0214	2.0427	3.4045
15	0.2618	0.1264	0.2528	0.5055	1.5165	3.0330	6.0661	10.1101
22.5	0.3927	0.1869	0.3737	0.7474	2.2423	4.4846	8.9691	14.9486
30	0.5236	0.2441	0.4883	0.9766	2.9297	5.8594	11.7188	19.5313
45	0.7854	0.3453	0.6905	1.3811	4.1432	8.2864	16.5728	27.6214
60	1.0472	0.4229	0.8457	1.6915	5.0744	10.1487	20.2975	33.8291

Finally, if we note that it takes 2 yds per pixel for detection, 0.5 yds per pixel for classification, and 0.25 yds per pixel for identification, we can apply these terms to the chart above and see which ranges fall within these limitations at various fields of view.

Field of View (Degrees)	**Slant Range (NM)**		
	Detect	**Classify**	**Identify**
1.5	30	10	5
5	12	3	2
15	4	1	0.5
22.5	3	0.6	0.3
30	2	0.5	0.25
45	1.5	0.3	0.15
60	1.2	0.25	0.1

The assumption that it takes 2 yds/pixel for detection, 0.5 yds/pixel for classification, and 0.25 yds/pixel for identification is made from referencing the following Web sites:

http://online.nps.navy.mil/imes/NIIR_IME.html;

http://www.fas.org/irp/imint/niirs_c/app3.htm.

APPENDIX III.B

Figure 33 shows that when comparing patterns for a single UAV with EO/IR sensor, no pattern is significantly better. Although for this scenario, no pattern breaks out as optimal, it was shown earlier in Figure 19 and later in Appendix III.C, that alternate is best.

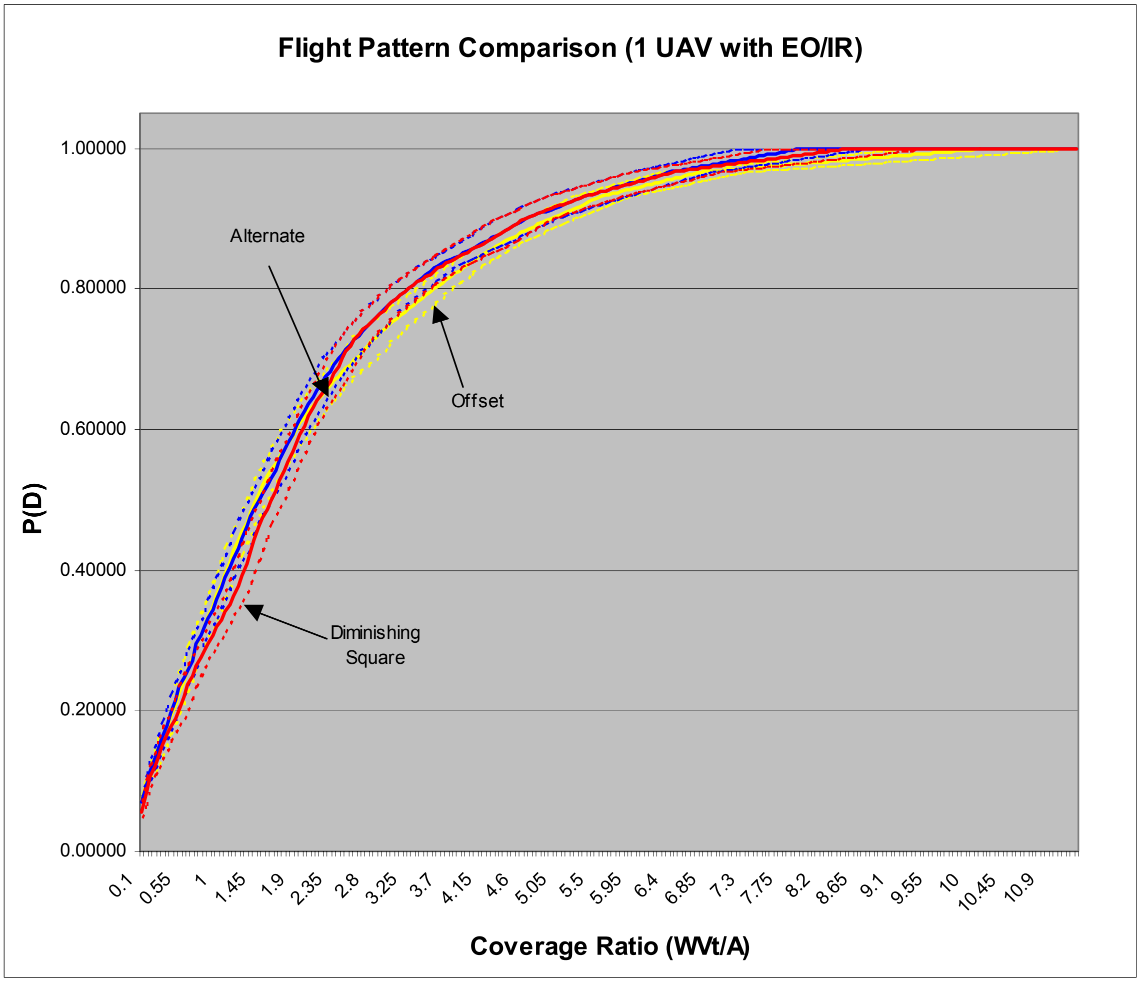

Figure 33. Flight Pattern Comparison (1 UAV with EO/IR).

APPENDIX III.C

Figure 34 is similar to Figure 19, which demonstrates that a UAV with a FLIR sensor has a higher P(D) versus Coverage Ratio when flying an alternate pattern vice a random search pattern.

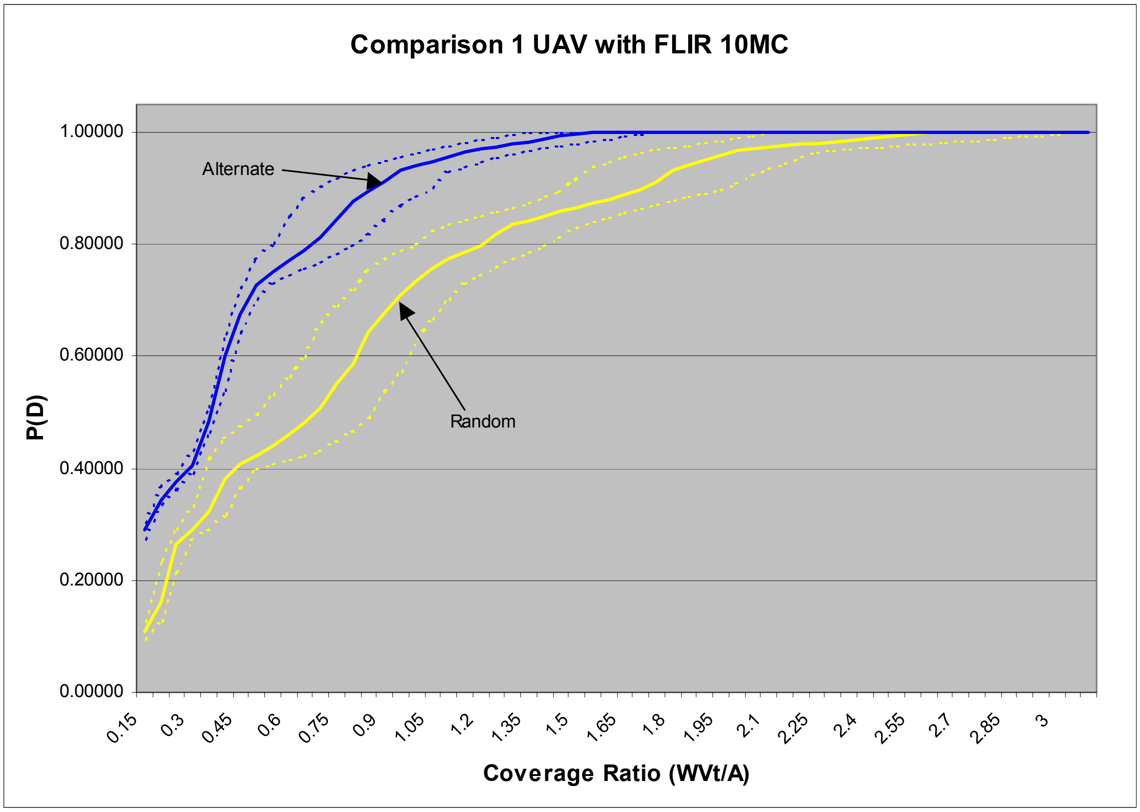

Figure 34. Comparison 1 UAV with FLIR random and lawnmower alternating pattern.

APPENDIX III.D

Figure 35 shows that when two UAVs fly in formation, they are significantly more effective when flying an alternate pattern vice an offset or diminishing squares pattern. As coverage ratio approaches 1.8, the alternate pattern becomes significantly better (no overlapping 95% confidence intervals), until a ratio of 2.5.

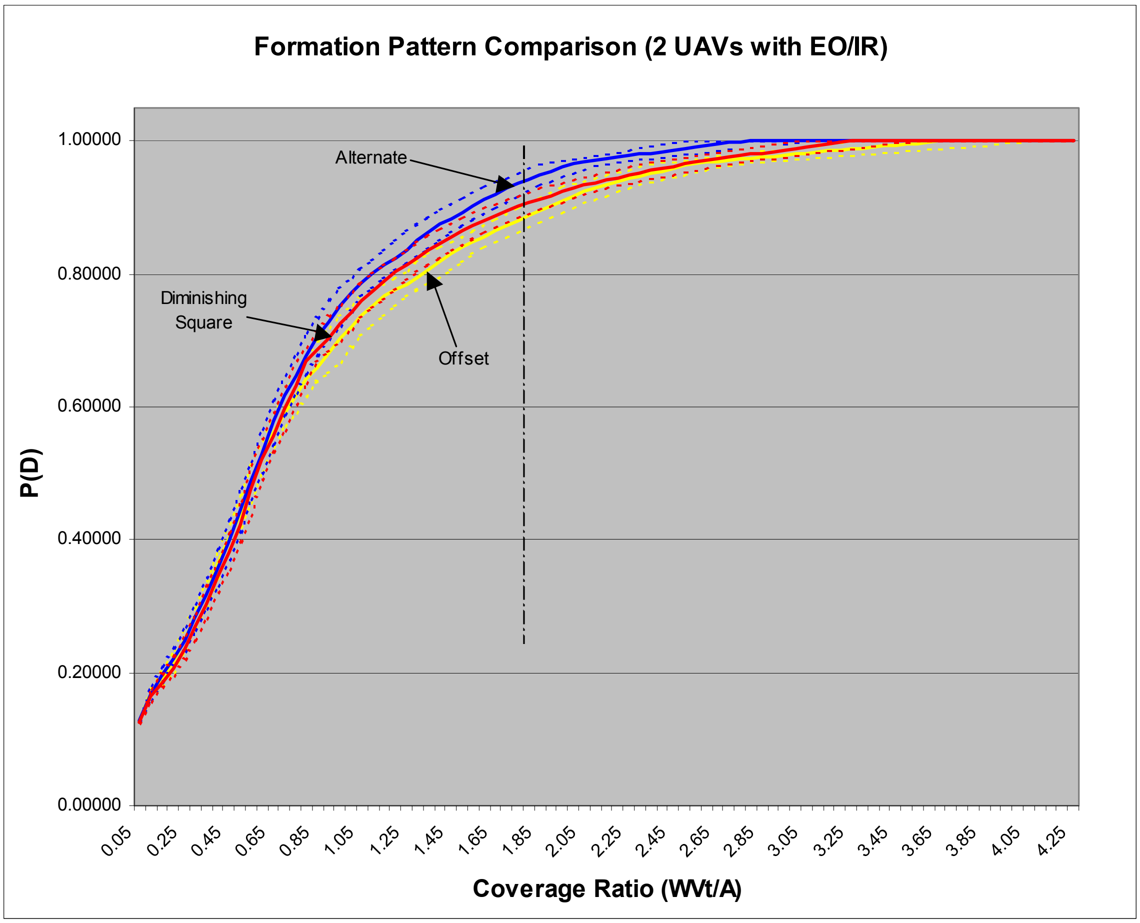

Figure 35. Formation Pattern Comparison (2 UAVs with EO/IR).

APPENDIX III.E

In this scenario, the 10 NM x 20 NM area was subdivided three ways: opposing, vertical split and horizontal split. In the opposing method, one UAV started at the southwest corner of the area (i.e., coordinate 0.0, 0.0), and the other UAV was started at the northeast corner (i.e., 10 NM, 20 NM coordinate). Then, the two UAVs flew their respective alternating pattern. The second method, vertical split, sectored the area vertically, and the third method, horizontal split, sectored the area in a horizontal manner. Figure 36 shows that vertically splitting the area is significantly better. Only in the high P(D) region is it about the same as the horizontal split method. Yet, overall, vertically splitting the area is more effective. Similar results were found when the number of UAVs was increased to three, four, and five UAVs.

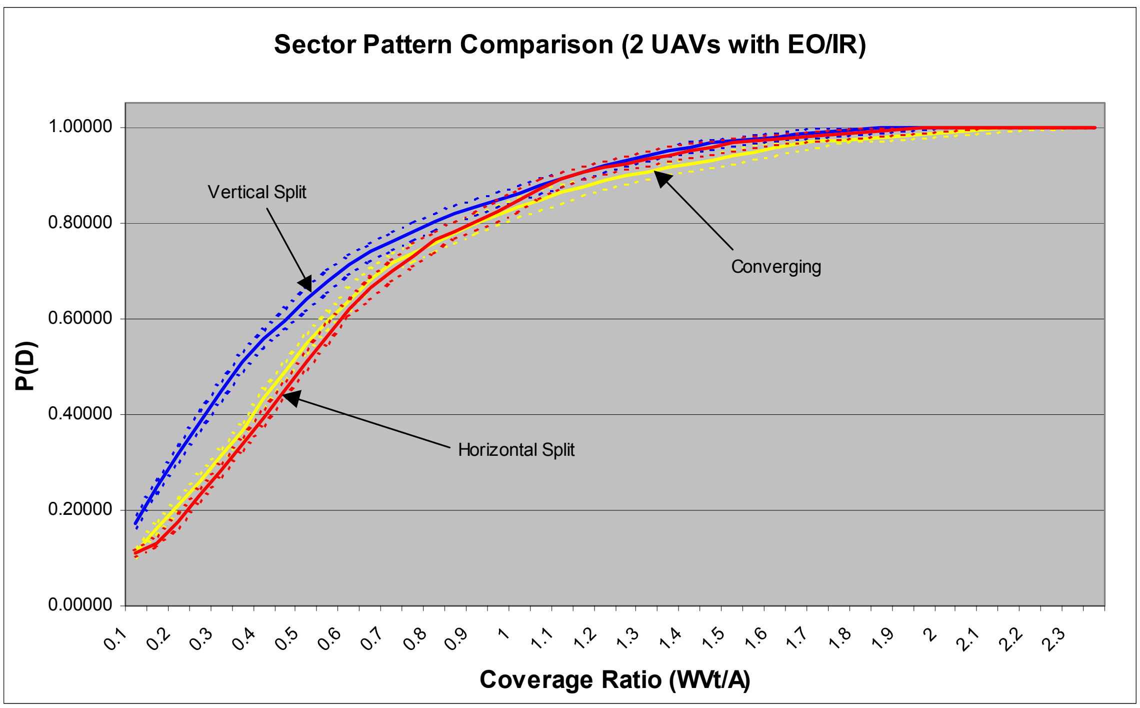

Figure 36. Two UAVs with EO/IR sensor subdividing the area in three ways.

APPENDIX III.F

Figure 37 shows that for two or four UAVs, sector with a vertical split and alternating pattern is significantly more effective than flying them in a formation alternating pattern. However, for three UAVs the outcome reverses. Looking further into five UAVs to determine if formation or sectoring would be better, the search area had to be doubled since with "cookie cutter" sensors, the five UAVs would only have to make one pass through the 10 NM x 20 NM area to detect all targets regardless of using formation or subdivision tactics. Five UAVs in formation proved to be significantly better than sector in this expanded area scenario.

For two or four UAVs sectors worked best, but for three or five UAVs, formations did. It is inconclusive to say whether one tactic is more effective over all. We can say, however, that tactics do matter.

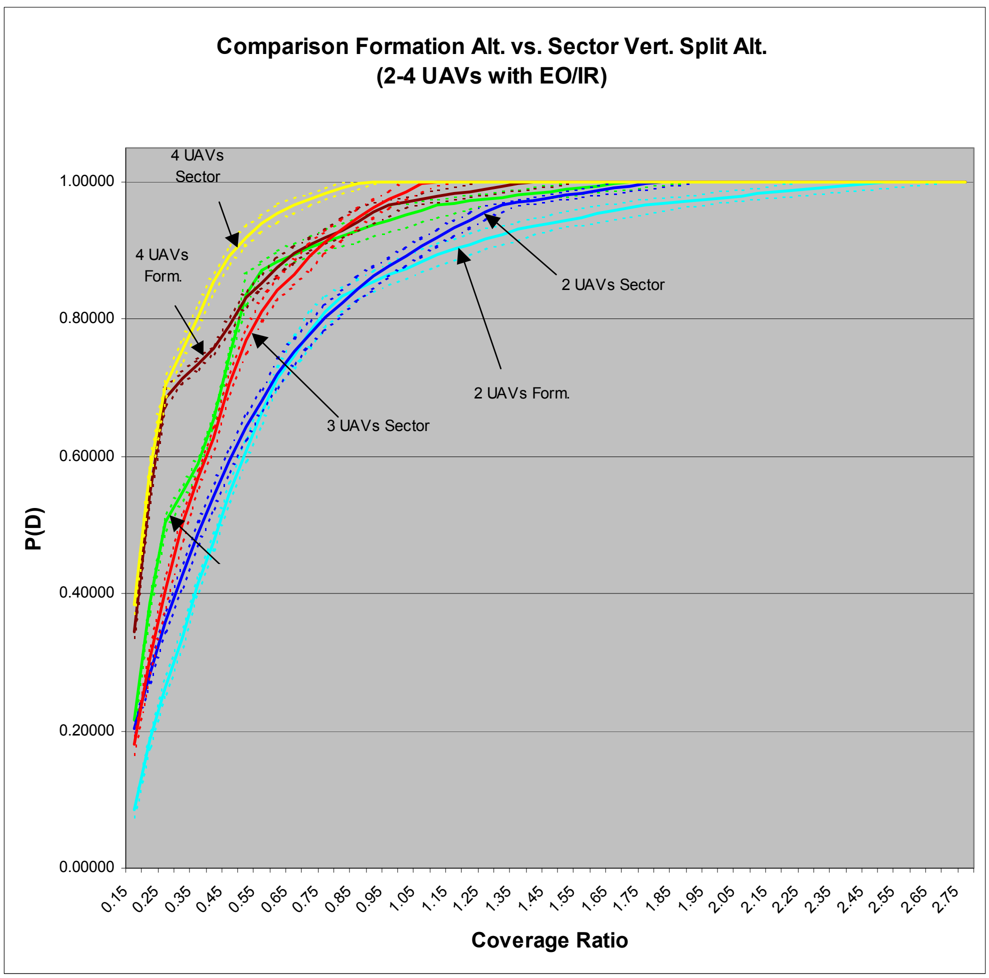

Figure 37. Comparison Formation Alt. vs. Vert. Split Alt. (2-4 UAVs with EO/IR).

APPENDIX III.G

Figure 38 shows that as more UAVs are added, P(D) improves until there are five UAVs in the limited search area. At that point, since we are using "cookie cutter" sensors, there is no additional gain in adding UAVs. Table 9 in Appendix III.H shows how much more effective it is to add additional UAVs to the area. In fact, the performance approximated that expected of an exhaustive search, further validating the quality of this simulation.

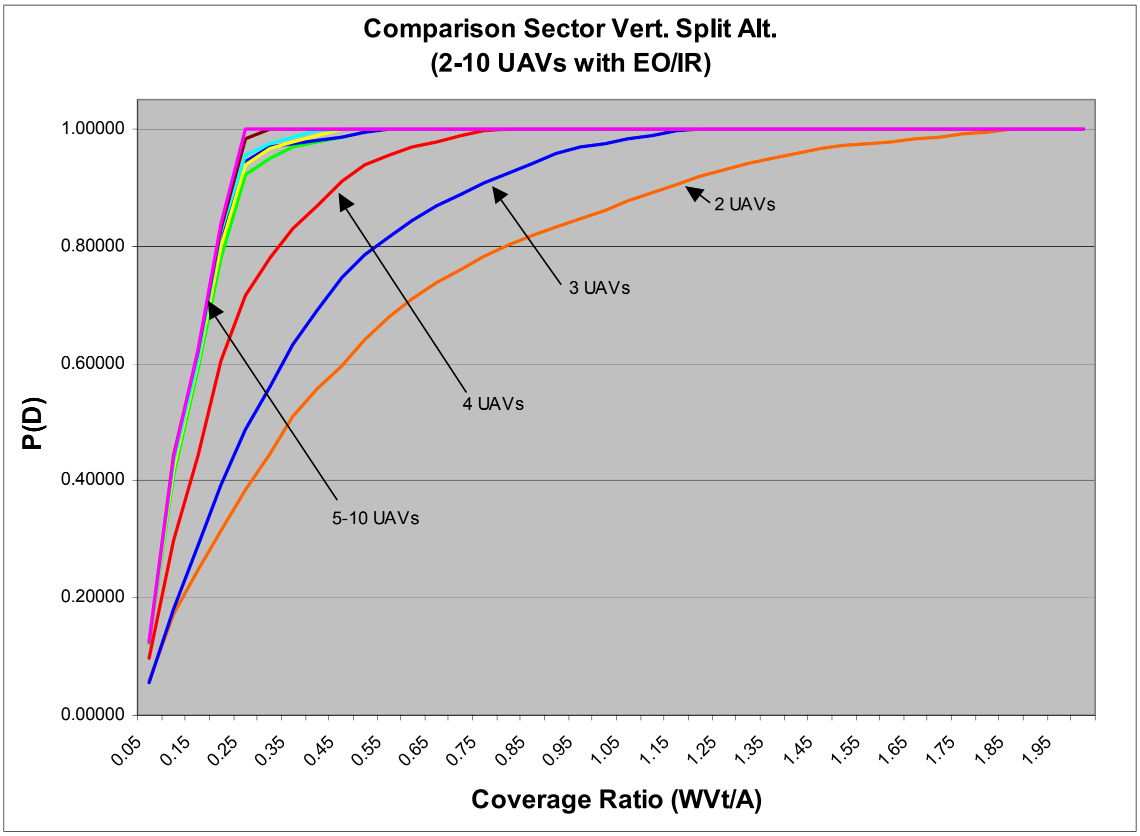

Figure 38. 2-10 UAVs vertical split alternating pattern comparison.

APPENDIX III.H

Table 9 shows that two UAVs increase the effectiveness of P(D) fourfold in the highlighted column on the left. In other words, the time required to achieve a certain probability of detection takes four times longer using one UAV than it does with two UAVs. Also, by adding UAVs, the effectiveness increases 1.3, 1.69, and 1.47, respectively. These results show that effectiveness increases greatly with the addition of a second UAV. Effectiveness also increases with the addition of a third, fourth, and fifth UAV highlighted in the three columns on the right; however, the increase is not as great.

	Factor Times Better Than One UAV					**Factor Times Better Than One Less UAV**				
# of UAVs	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
P(D)										
0.10	1.00	2.37	1.91	3.13	4.26	1.00	2.37	0.80	1.64	1.36
0.20	1.00	4.04	3.25	6.15	7.55	1.00	4.04	0.80	1.89	1.23
0.30	1.00	4.05	4.25	7.60	9.26	1.00	4.05	1.05	1.79	1.22
0.40	1.00	4.32	5.55	8.43	11.47	1.00	4.32	1.29	1.52	1.36
0.50	1.00	4.42	6.15	9.01	12.34	1.00	4.42	1.39	1.46	1.37
0.60	1.00	4.24	5.34	9.73	12.59	1.00	4.24	1.26	1.82	1.29
0.70	1.00	4.21	5.73	10.12	13.69	1.00	4.21	1.36	1.77	1.35
0.80	1.00	4.16	6.91	10.32	15.98	1.00	4.16	1.66	1.49	1.55
0.87	1.00	4.14	8.04	10.61	19.07	1.00	4.14	1.94	1.32	1.80
0.90	1.00	4.18	7.01	10.85	20.50	1.00	4.18	1.68	1.55	1.89
0.97	1.00	4.44	5.88	11.01	18.78	1.00	4.44	1.32	1.87	1.71
1.00	1.00	4.32	4.74	10.49	15.34	1.00	4.32	1.10	2.21	1.46
AVG	—	4.07	—	—	—	—	—	1.30	1.69	1.47

Table 9. Increases in effectiveness for multiple UAVs.

Note: Times that were used to generate this table resulted from Sector Vertical Split Alternating pattern except for column 3. Three UAVs were shown to be more effective in a 10 NM x 20 NM area flying a Formation Alternating pattern. Therefore, those times were used to generate column 3.

APPENDIX IV.A – THE EO SENSOR FOR THE INFLUENCE DIAGRAMS

Midgrade quality sensor:

Target	Real				Nothing			
Day Night	Day		Night		Day		Night	
Weather	Good	Bad	Good	Bad	Good	Bad	Good	Bad
No Target	0.1	0.2	0.3	0.6	0.9	0.75	0.65	0.45
Maybe	0.1	0.2	0.2	0.2	0.1	0.2	0.25	0.35
Sure	0.8	0.6	0.5	0.2	0	0.05	0.1	0.2

Good quality sensor:

Target	Real				Nothing			
Day Night	Day		Night		Day		Night	
Weather	Good	Bad	Good	Bad	Good	Bad	Good	Bad
No Target	0.05	0.1	0.1	0.2	0.95	0.9	0.85	0.75
Maybe	0.1	0.2	0.2	0.3	0.05	0.05	0.1	0.15
Sure	0.85	0.7	0.7	0.5	0	0.05	0.05	0.1

Low quality sensor:

Target	Real				Nothing			
Day Night	Day		Night		Day		Night	
Weather	Good	Bad	Good	Bad	Good	Bad	Good	Bad
No Target	0.2	0.25	0.25	0.3	0.85	0.8	0.6	0.4
Maybe	0.2	0.2	0.25	0.3	0.15	0.15	0.3	0.4
Sure	0.6	0.55	0.5	0.4	0	0.05	0.1	0.2

APPENDIX IV.B – SENSITIVITY ANALYSIS

Number of Sensors	#	Rules	P(detection)	P(false alarm)
1	1	Any detection	0.91	0.1
	2	Certain detections only	0.74	0.03
2	3	Any detection	0.99	0.18
	4	Any detection by at least two sensors	0.83	0.01
	5	At least one certain detection	0.92	0.05
	6	At least one certain detection and one possible detection	0.79	0.01
	7	Two certain detections	0.55	0
	8	One certain detection or two possible detections	0.95	0.06
3	9	Any detection	1	0.26
	10	Any detection by at least two sensors	0.97	0.04
	11	Any detection by three sensors	0.77	0
	12	At least one certain detection	0.97	0.08
	13	One certain detection or two possible detections	0.99	0.09
	14	One certain detection or three possible detections	0.98	0.08
	15	At least one certain detection and one possible detection	0.95	0.02
	16	At least one certain detection and two possible detections	0.75	0
	17	At least two certain detections	0.81	0
	18	At least two certain detections or three possible detections	0.89	0.01
	19	At least two certain detections and one possible detection	0.68	0
	20	Three certain detections	0.42	0

Table 10. Rules and results for a good sensor.

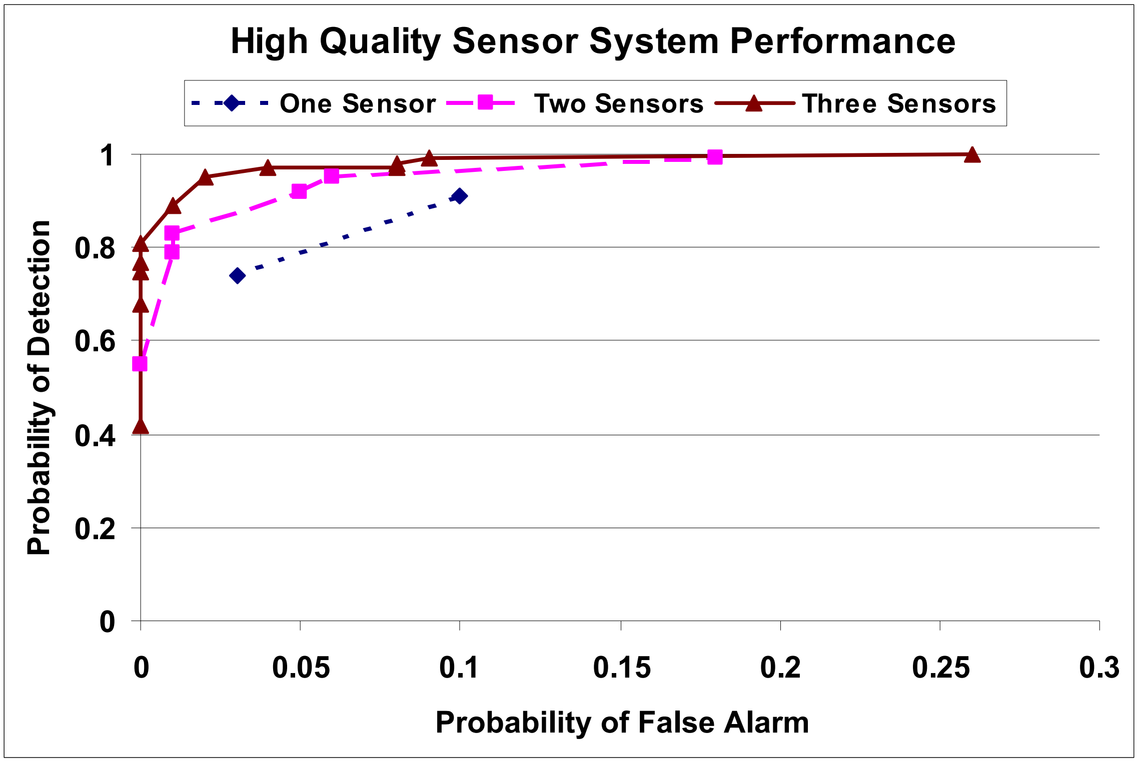

Figure 39. Pd vs. Pf for a good sensor.

Number of Sensors	#	Rules	P(detection)	P(false alarm)
1	1	Any detection	0.76	0.26
	2	Certain detections only	0.53	0.05
2	3	Any detection	0.94	0.43
	4	Any detection by at least two sensors	0.58	0.09
	5	At least one certain detection	0.78	0.09
	6	At least one certain detection and one possible detection	0.53	0.04
	7	Two certain detections	0.29	0.01
	8	One certain detection or two possible detections	0.83	0.14
3	9	Any detection	0.99	0.55
	10	Any detection by at least two sensors	0.86	0.19
	11	Any detection by three sensors	0.48	0.04
	12	At least one certain detection	0.89	0.13
	13	One certain detection or two possible detections	0.95	0.24
	14	One certain detection or three possible detections	0.91	0.14
	15	At least one certain detection and one possible detection	0.8	0.08
	16	At least one certain detection and two possible detections	0.44	0.02
	17	At least two certain detections	0.55	0.02
	18	At least two certain detections or three possible detections	0.65	0.05
	19	At least two certain detections and one possible detection	0.35	0.01
	20	Three certain detections	0.16	0

Table 11. Rules and results for a low quality sensor.

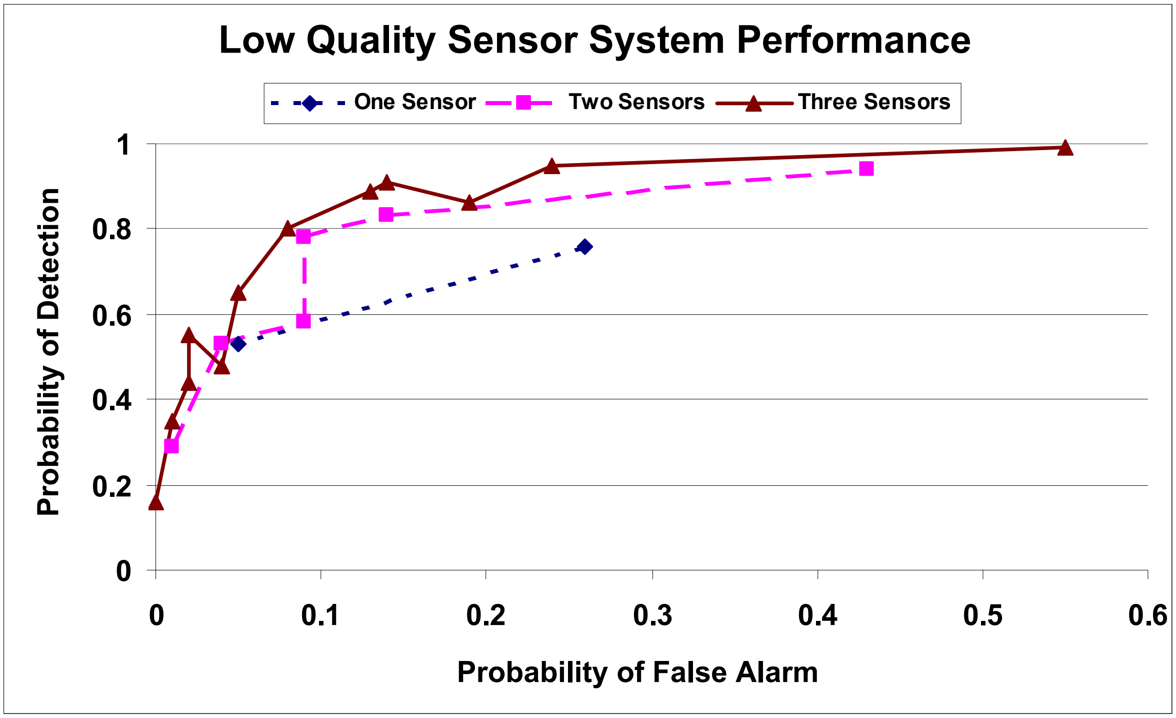

Figure 40. Pd vs. Pf for a low quality sensor.

APPENDIX IV.C – COMPARING QUALITY AND QUANTITY

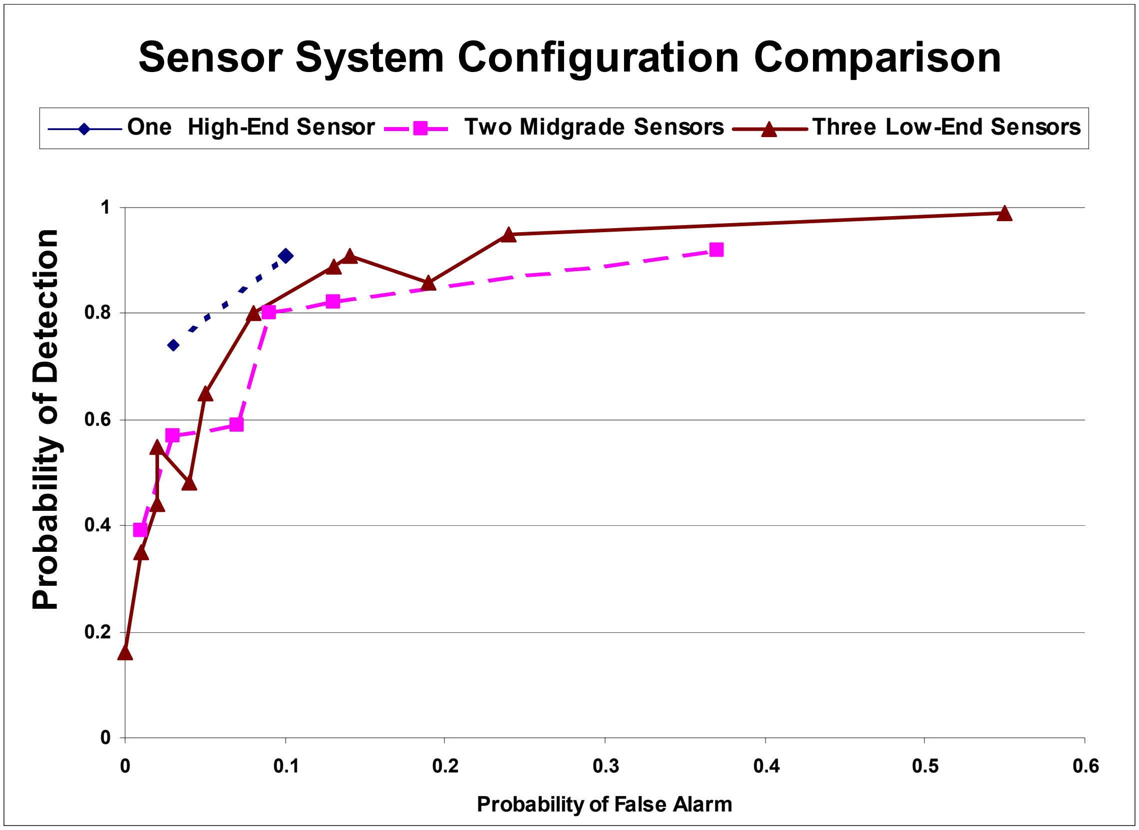

Figure 41. A comparison of a single good sensor to a two midgrade sensors system, to a three low grade sensor system.

INITIAL DISTRIBUTION LIST

1. Research Office (Code 09)..1
   Naval Postgraduate School
   Monterey, CA 93943-5000

2. Dudley Knox Library (Code 013)..2
   Naval Postgraduate School
   Monterey, CA 93943-5002

3. Defense Technical Information Center...2
   8725 John J. Kingman Rd., STE 0944
   Ft. Belvoir, VA 22060-6218

4. Richard Mastowski (Editorial Assistant)..2
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA 93943-5219

5. LCDR Russell Gottfried 6
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA 93943-5219

6. Professor James N. Eagle..1
   Chairman, Department of Operations Research
   Naval Postgraduate School
   Monterey, CA 93943-5219

7. Senior Lecturer David Olwell...1
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA 93943-5219

8. CAPT Jeffrey Kline 1
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA 93943-5219

9. Senior Lecturer Thomas H. Hoivik...2
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA 93943-5219